FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 8

Topic: The Torsion Formula , Non-uniform torsion, Stresses and strains in pure shear

Instructor:

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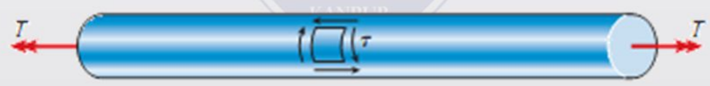
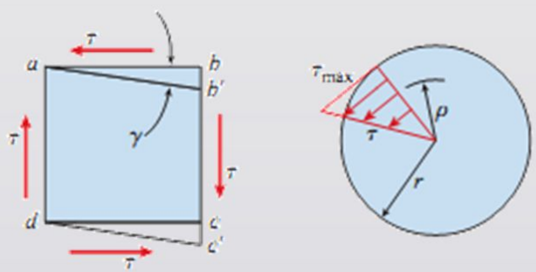
The Torsion Formula

- ✓ In this lecture we will determine the relationship between the shear stresses and the torque T.
- ✓ The distribution of the shear stresses acting on a cross section is pictured in below Because these stresses act continuously around the cross section, they have a resultant in the form of a moment—a moment equal to the torque T acting on the bar



(a)



The Torsion Formula

- ✓ To determine this resultant, we con-sider an element of area $\boldsymbol{dA}$ located at radial distance r from the axis of the bar . The shear force acting on this element is equal to $\boldsymbol{\tau dA}$, where $\boldsymbol{\tau}$ is the shear stress at radius r.
- ✓ The moment of this force about the axis of the bar is equal to the force times its distance from the center, or $\boldsymbol{\tau \rho dA}$.
- ✓ There elemental moment is

$$dM = \tau dA = \frac{\tau_{max}}{r} \rho^2 dA$$

The resultant moment (torque T) is

$$T = \int_A dM = \frac{\tau_{max}}{r} \int_A \rho^2 dA = \frac{\tau_{max}}{r} \mathrm{I_P}$$

Where$\mathrm{I_P} = \int_A \rho^2 dA\,,\ is\ polar\ moment\ of\ inertia$

- ✓ Polar moments of inertia have units of length to the fourth power

Max Shear Stress

- ✓ An expression for the maximum shear stress is given by

$$\tau_{max} = \frac{Tr}{I_p}$$

- ✓ For a prismatic bars of solid circular cross section

$$r = \frac{d}{2} \text{ and } I_P = \frac{\pi d^4}{32}$$

$$\tau_{max} = \frac{16T}{\pi d^3}$$

- ✓ Shear Stress at a distance of $\boldsymbol{r}$ from the center of bar is given by

$$\tau = \frac{\rho}{r}\tau_{max} = \frac{T\rho}{I_p}$$

Angle of Twist

- ✓ The angle of twist of a bar of linearly elastic material can now be related to the applied torque T

$$\theta = \frac{T}{GI_p}$$

$GI_p \text{ is known as Torsional Rigidity}$

$$\phi = \frac{TL}{GI_p}$$

$\frac{GI_p}{L} = k_T \text{ is called } \textbf{torsional stiffness}$

θ has units of radians per unit of length

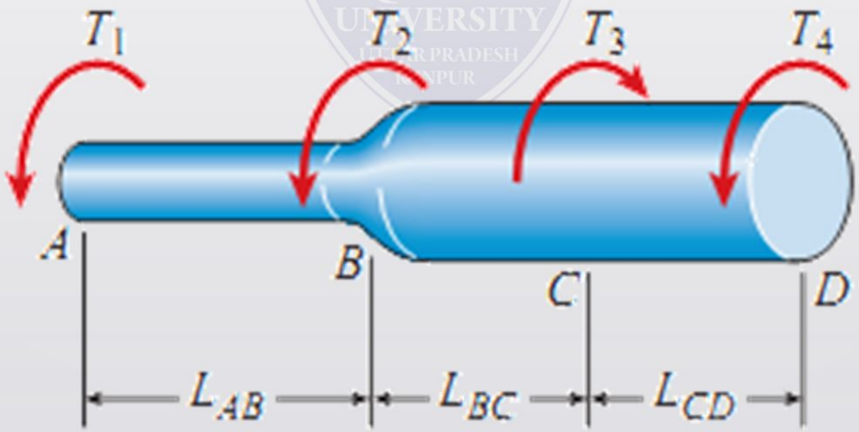
- ✓ Equations for the torsion of circular bars and tubes ***cannot be used for bars of other shapes***. Noncircular bars, such as rectangular bars and bars having I-shaped cross sections, their cross sections do not remain plane and their maximum stresses are not located at the farthest distances from the midpoints of the cross sections

Non-uniform torsion

Case 1: Bar consisting of prismatic segments with constant torque throughout each segment

$$\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \cdots + \phi_n$$

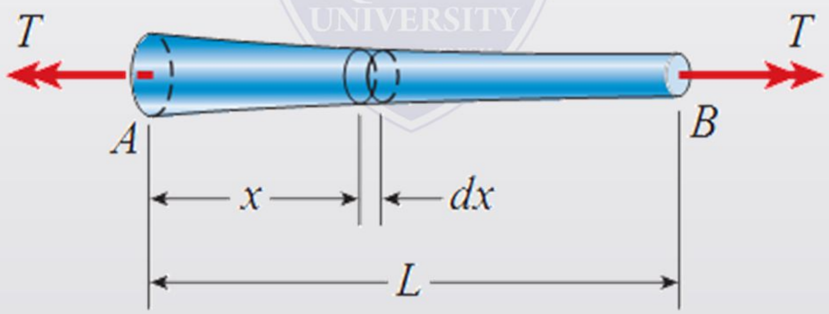
$$\phi = \sum_{i=1}^{n} \phi_i = \sum_{i=1}^{n} \frac{T_i L_i}{G_i (I_p)_i}$$



Non-uniform torsion

Case 2: Bar with continuously varying cross sections and constant torque

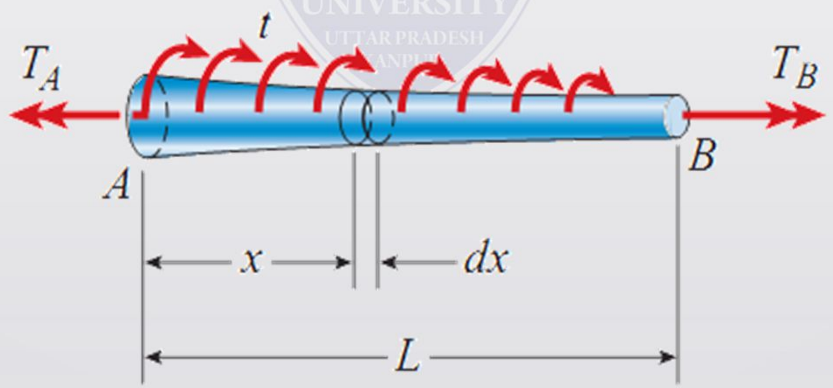
$$\phi = \int_0^L d\phi = \int_0^L \frac{T dx}{G I_p(x)}$$



Non-uniform torsion

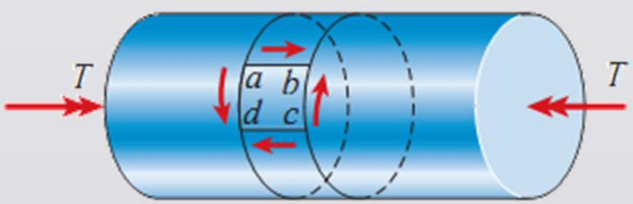
Case 3: Bar with continuously varying cross sections and continuously varying torque

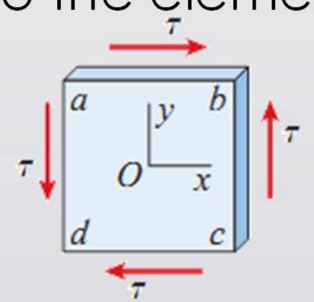
$$\phi = \int_0^L d\phi = \int_0^L \frac{T(x)dx}{GI_p(x)}$$



Stresses and Strain in Pure Shear

- ✓ by considering a stress element abcd cut between two cross sections of a bar in torsion . This element is in a state of pure shear, because the only stresses acting on it are the shear stresses τ on the four side face
- ✓ The directions of these shear stresses depend upon the directions of the applied torques T
- ✓ This same state of stress exists for a similar element cut from the interior of the bar, except that the magnitudes of the shear stresses are smaller because the radial distance to the element is smaller



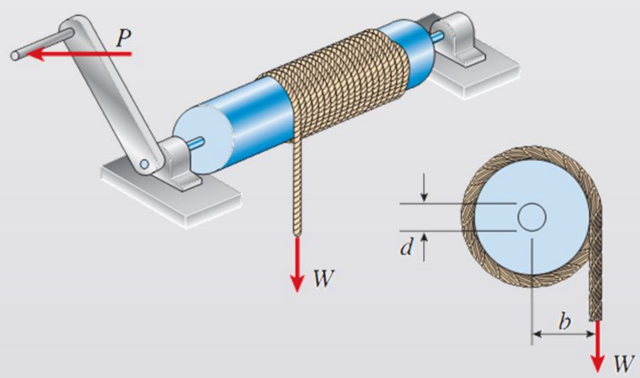


Question

3.3-1 A prospector uses a hand-powered winch (see figure) to raise a bucket of ore in his mine shaft. The axle of the winch is a steel rod of diameter $d = 15$ mm. Also, the distance from the center of the axle to the center of the lifting rope is $b = 100$ mm.

(a) If the weight of the loaded bucket is $W = 400$ N, what is the maximum shear stress in the axle due to torsion?

(b) If the maximum bucket load is 510 N and the allowable shear stress in the axle is 65 MPa, what is the minimum permissible axle diameter?

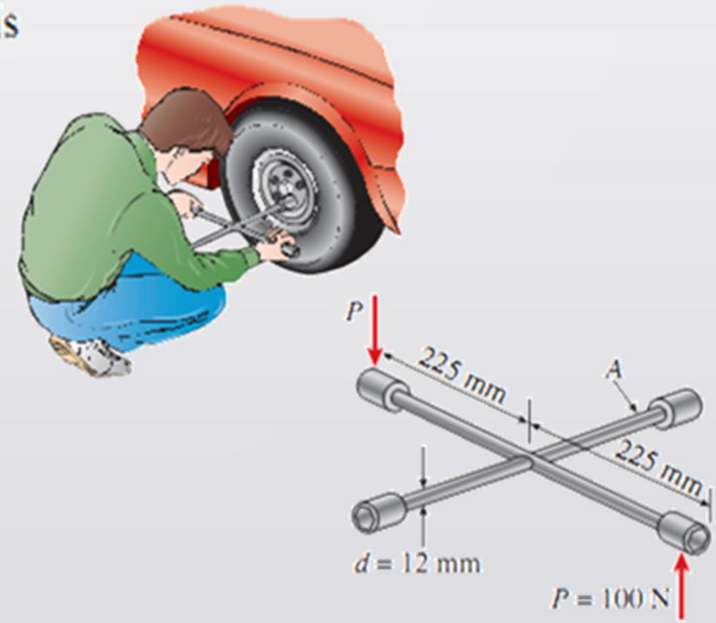


Question

3.3-3 While removing a wheel to change a tire, a driver applies forces $P = 100$ N at the ends of two of the arms of a lug wrench (see figure). The wrench is made of steel with shear modulus of elasticity $G = 78$ GPa. Each arm of the wrench is 255 mm long and has a solid circular cross section of diameter $d = 12$ mm.

(a) Determine the maximum shear stress in the arm that is turning the lug nut (arm A).

(b) Determine the angle of twist (in degrees) of this same arm.

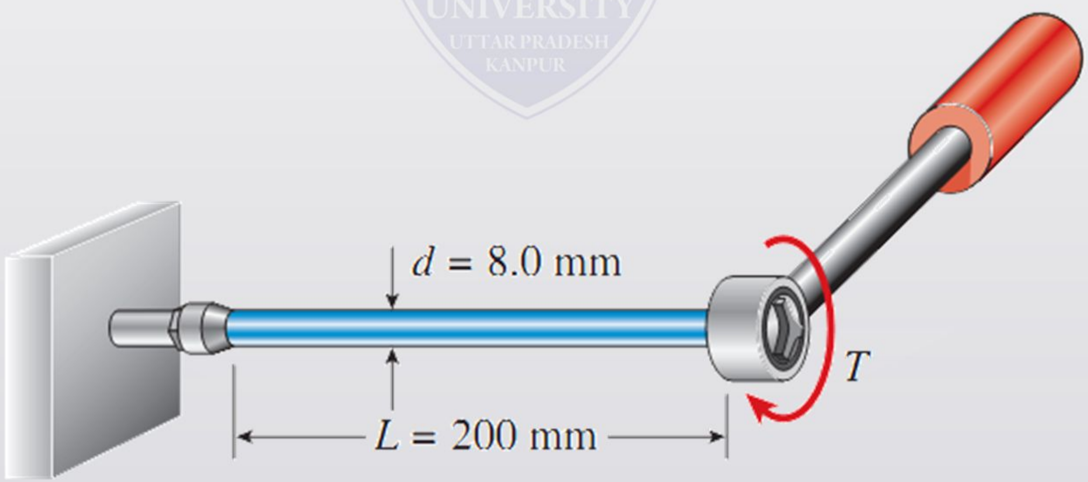


Question

3.3-6 The steel shaft of a socket wrench has a diameter of 8.0 mm. and a length of 200 mm (see figure).

If the allowable stress in shear is 60 MPa, what is the maximum permissible torque T_{max} that may be exerted with the wrench?

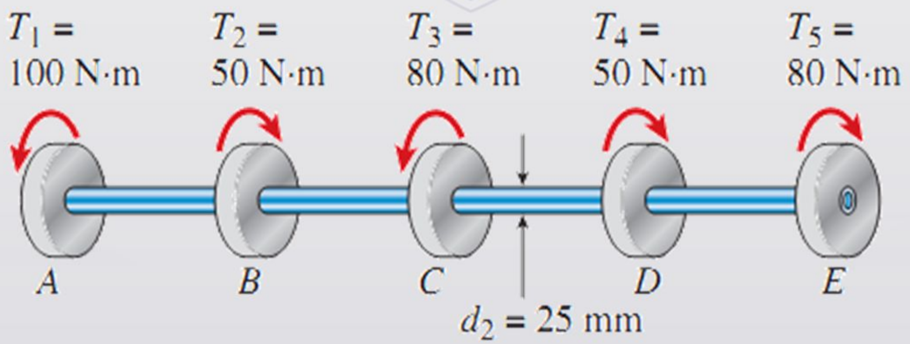
Through what angle ϕ (in degrees) will the shaft twist under the action of the maximum torque? (Assume $G = 78$ GPa and disregard any bending of the shaft.)



Question

3.4-5 A hollow tube *ABCDE* constructed of monel metal is subjected to five torques acting in the directions shown in the figure. The magnitudes of the torques are $T_1 = 100$ N·m, $T_2 = T_4 = 50$ N·m, and $T_3 = T_5 = 80$ N·m. The tube has an outside diameter of $d_2 = 25$ mm. The allowable shear stress is 80 MPa and the allowable rate of twist is 6°/m.

Determine the maximum permissible inside diameter d_1 of the tube.



Question

3.4-8 A tapered bar AB of solid circular cross section is twisted by torques T (see figure). The diameter of the bar varies linearly from d_A at the left-hand end to d_B at the right-hand end.

(a) Confirm that the angle of twist of the tapered bar is

$$\phi = \frac{32TL}{3\pi G(d_B - d_A)}\left(\frac{1}{d_A^3} - \frac{1}{d_B^3}\right)$$

(b) For what ratio d_B/d_A will the angle of twist of the tapered bar be one-half the angle of twist of a prismatic bar of diameter d_A? (The prismatic bar is made of the same material, has the same length, and is subjected to the same torque as the tapered bar.)