FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 9

Topic:9. State of Plane stress, Stresses on Inclined Sections, Transformation Equations for Plane Stress

Instructor:

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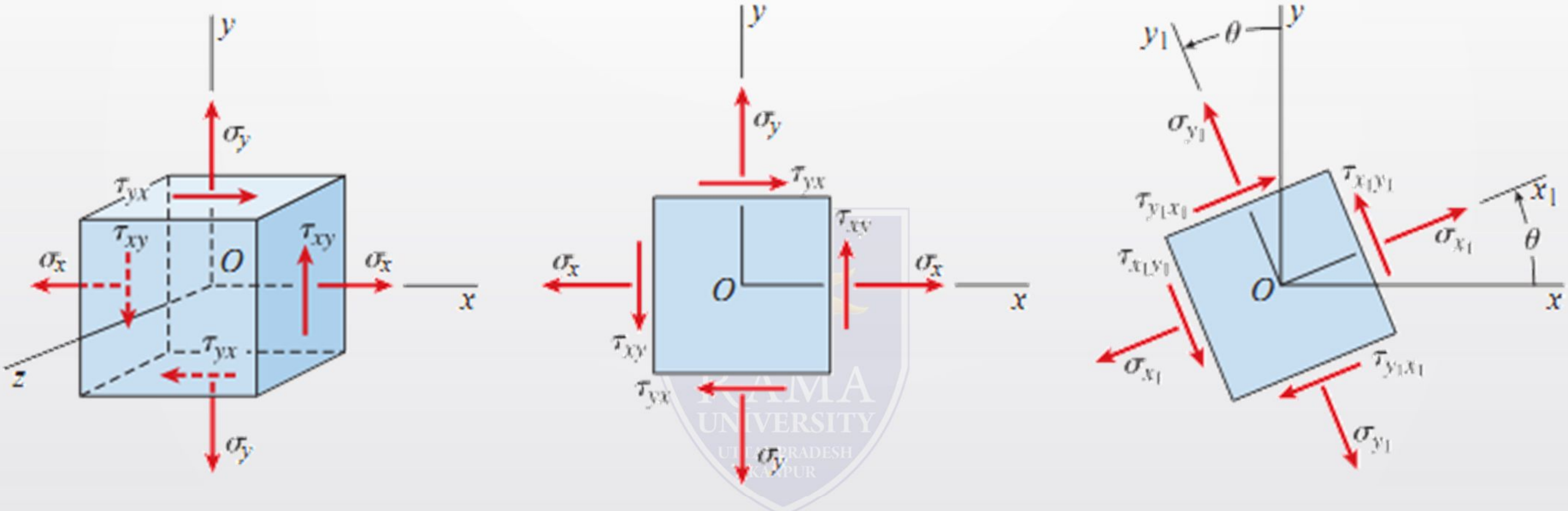
Stresses

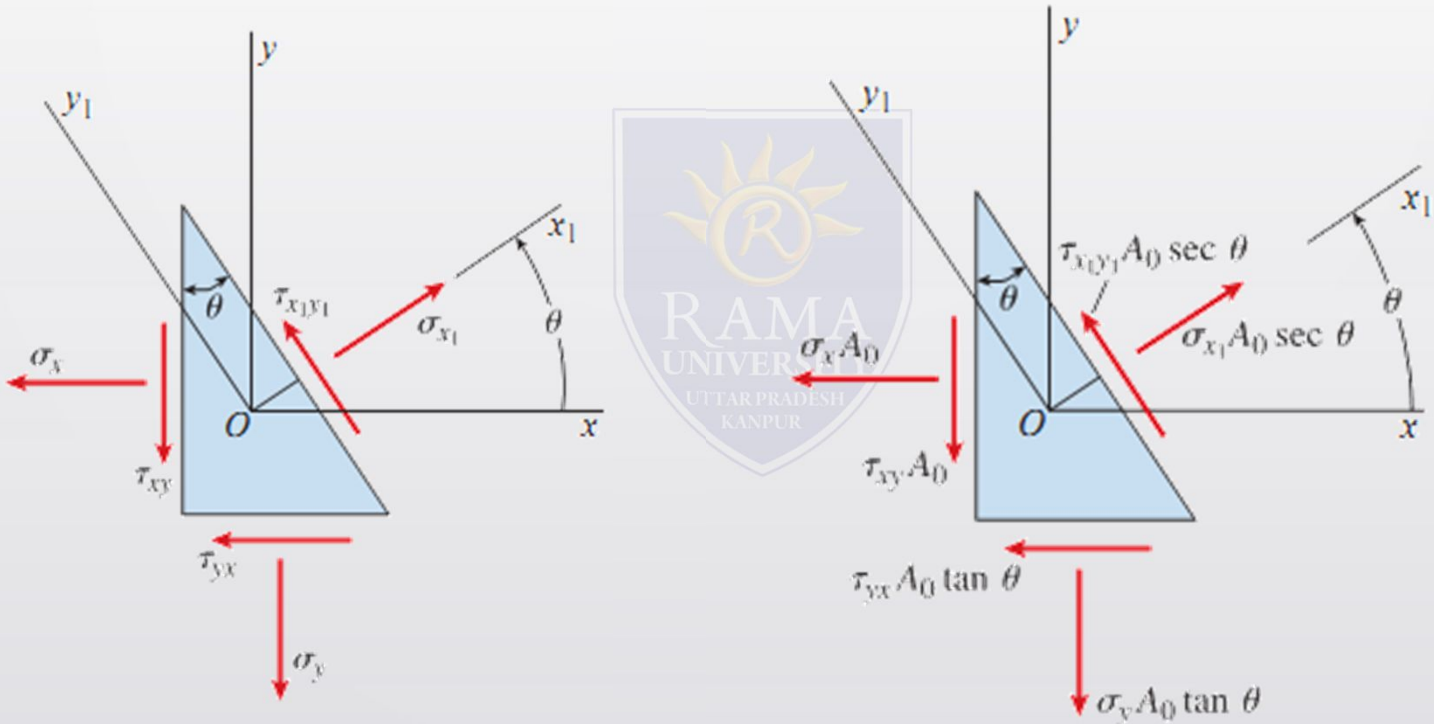
- ✓ A normal stress s has a subscript that identifies the face on which the stress acts; for instance, the stress σ_x acts on the x face of the element and the stress σ_y acts on the y face of the element.
- ✓ A shear stress t has two subscripts—the first subscript denotes the face on which the stress acts, and the second gives the direction on that face. Thus, the stress τ_{xy} acts on the x face in the direction of the y axis

Stresses on Inclined Sections



- ✓ Elements in plane stress: (a) three-dimensional view of an element oriented to the xyz axes, (b) two-dimensional view of the same element, and (c) two-dimensional view of an element oriented to the $x_1y_1z_1$ axes

Stresses on Inclined Sections



Stresses on Inclined Sections

- ✓ The stresses acting on the inclined $\boldsymbol{x_1 y_1}$ element can be expressed in terms of the stresses on the **xy** element by using equations of equilibrium
- ✓ Let us denote the area of the left-hand side face (that is, the negative x face) as A_0
- ✓ Summing forces in the $\boldsymbol{x_1}$ and $\boldsymbol{y_1}$ direction we will get the transformation equations for plane stress

Along $\mathbf{x_1}$

$$\sigma_{X1}A_0\sec\theta-\sigma_X A_0\cos\theta-\tau_{xy}A_0\sin\theta-\sigma_y A_0\tan\theta\sin\theta-\tau_{xy}A_0\tan\theta\cos\theta=0$$

Along $\mathbf{y_1}$

$$\tau_{x1y1}A_0\sec\theta+\sigma_X A_0\sin\theta-\tau_{xy}A_0\cos\theta-\sigma_y A_0\tan\theta\cos\theta+\tau_{xy}A_0\tan\theta\sin\theta=0$$

Stress Transformation Equations

- ✓ Using relationship $\tau_{xy} = \tau_{yx}$ and simplifying above equations we get

$$\sigma_{x1} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$

$$\tau_{x1y1} = -(\sigma_x - \sigma_y) \sin\theta \cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

- ✓ Using geometrical identities we can rearrange the above equations into

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

- ✓ Transformation equations were derived solely from equilibrium of an element, they are applicable to stresses in any kind of material, whether linear or nonlinear, elastic or inelastic

Stress Transformation Equations

- ✓ σ_{y_1} can be obtained by putting θ as $\theta + 90^o$

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

- ✓ Only one intrinsic ***state of stress*** exists at a point in a stressed body, regardless of the orientation of the element being used to portray that state of stress. When we have two elements with different orientations at the same point in a body, the ***stresses acting on the faces of the two elements are different, but they still represent the same state of stress***, namely, the stress at the point under consideration. This situation is analogous to the representation of a force vector by its components— although the components are different when the coordinate axes are rotated to a new position, the force itself is the same.

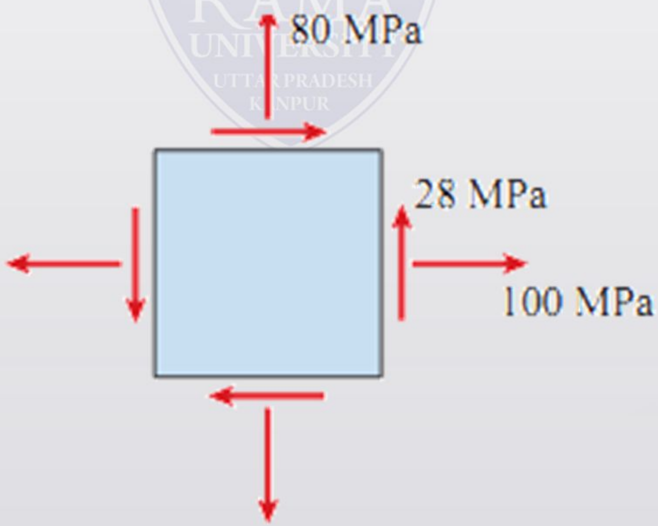
Stress Transformation Equations

$$\sigma_{x_1} + \sigma_{y_1} = \sigma_x + \sigma_y$$

- ✓ The above equation can be obtained by adding $\boldsymbol{\sigma_{x_1}}$ ***and*** $\boldsymbol{\sigma_{y_1}}$
- ✓ The shear stresses acting on all four side faces of an element in plane stress are known if we determine the shear stress acting on any one of those faces.

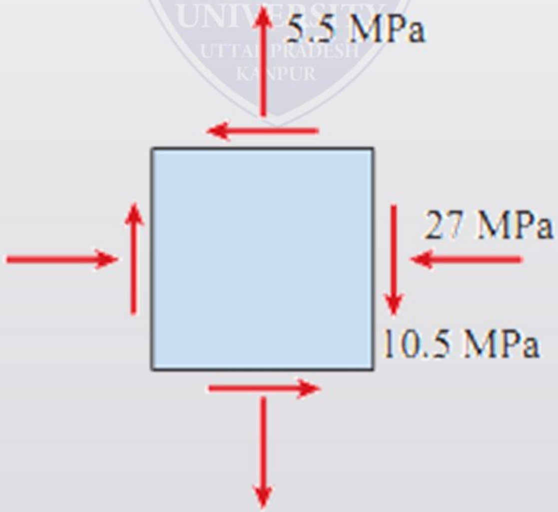
Questions

Q1 Solve the preceding problem for an element in plane stress subjected to stresses σ_x = 100 MPa, σ_y = 80 MPa, and τ_{xy} = 28 MPa, as shown in the figure. Determine the stresses acting on an element oriented at an angle θ=30° from the x axis, where the angle θ is positive when counterclockwise. Show these stresses on a sketch of an element oriented at the angle θ



Questions

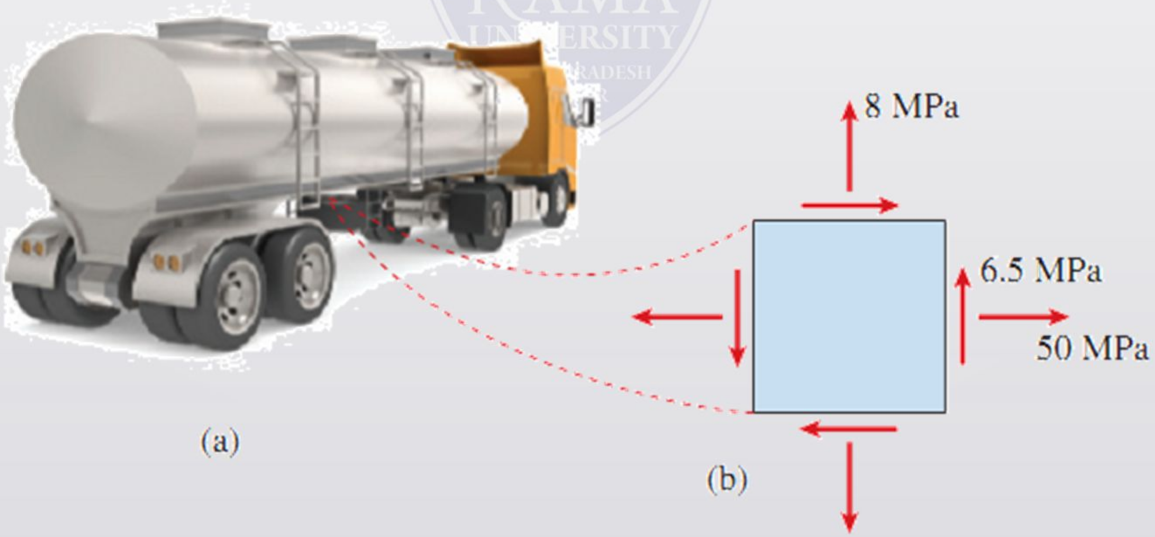
Q2 An element in plane stress from the fuselage of an air-plane is subjected to compressive stresses of magnitude 27 MPa in the horizontal direction and tensile stresses of magnitude 5.5 MPa in the vertical direction (see figure). Also, shear stresses of magnitude 10.5 MPa act in the directions shown. Determine the stresses acting on an element oriented at a clockwise angle of 35° from the horizontal. Show these stresses on a sketch of an element oriented at this angle.



Questions

7.2-1 The stresses on the bottom surface of a fuel tanker (figure part a) are known to be $\sigma_x = 50$ MPa, $\sigma_y = 8$ MPa, and $\tau_{xy} = 6.5$ MPa (figure part b).

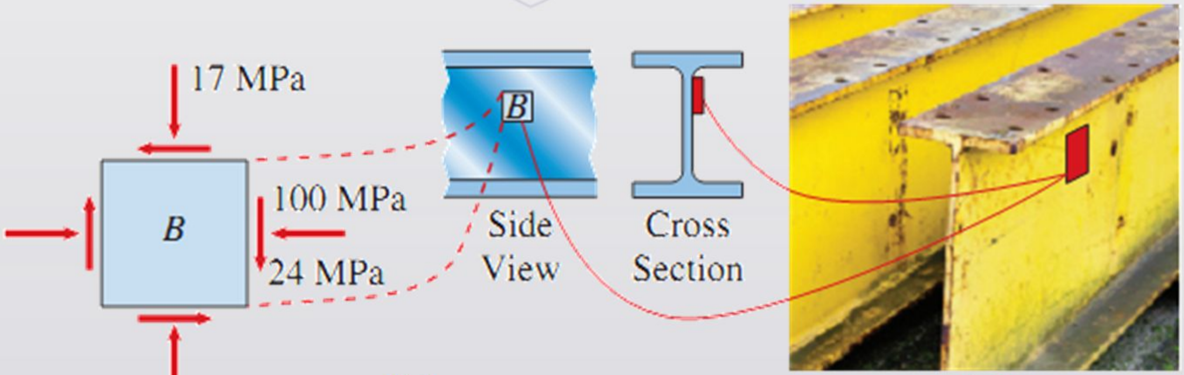
Determine the stresses acting on an element oriented at an angle $\theta = 52°$ from the x axis, where the angle θ is positive when counterclockwise. Show these stresses on a sketch of an element oriented at the angle θ.



Questions

7.2-7 The stresses acting on element B (see figure part a) on the web of a wide-flange beam are found to be 100 MPa in compression in the horizontal direction and 17 MPa in compression in the vertical direction (see figure part b). Also, shear stresses with a magnitude of 24 MPa act in the directions shown.

Determine the stresses acting on an element oriented at a counterclockwise angle of 36° from the horizontal. Show these stresses on a sketch of an element oriented at this angle.

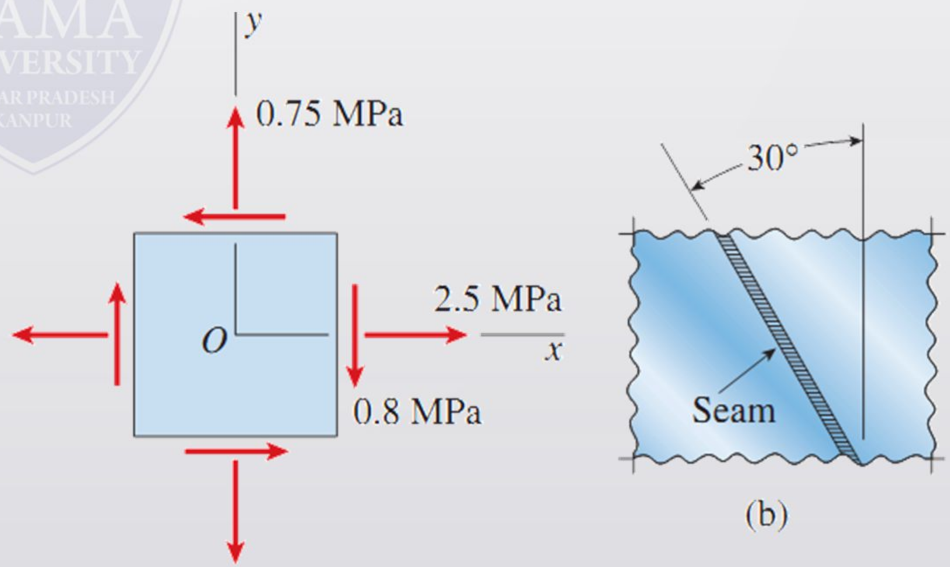


(b) (a)

Questions

7.2-9 The polyethylene liner of a settling pond is subjected to stresses $\sigma_x = 2.5$ MPa, $\sigma_y = 0.75$ MPa, and $\tau_{xy} = -0.8$ MPa, as shown by the plane-stress element in the figure part a.

Determine the normal and shear stresses acting on a seam oriented at an angle of 30° to the element, as shown in the figure part b. Show these stresses on a sketch of an element having its sides parallel and perpendicular to the seam.



(b)