Lecture No 15 Topic: Derivation

The equation of Line is: Y=mx+c ....(1)

Here m is the slope of Line and C is y-Intercept of line. The slope can also be written as

$$\frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{\text{change in } x}{\text{change in } y} \text{ or } \frac{dx}{dy} \quad .....(2)$$

Fir[illegible]ope is less than 1, then X will always be incremented. so at (m<=1)

we calculate the d1 which is distance between intersection point y to the pixel yk

So, d1 = y-yk

d1 = mxk+1 +c -yk by using ....(1)

Similarly d2 is the distance between pixel yk+1 and intersection point y.

d2 = yk+1-y

d2 = yk+1-(mxk+1+c) by using ....(1)

Note: here x is always incrementing so we can write xk+1 as xk +1 and here yk+1 is next pixel so we can write it as yk+1.

subtracting d2 from d1

d1-d2 = m(xk+1) +c -yk – [yk+1-(mxk+1+c)]

= m(xk+1)+c -yk – yk-1+m(xk+1)+c d1-d2

= 2m(xk+1)-2yk+2c-1 .....(3)

Derivation

$$d_1 - d_2 = 2\frac{dy}{dx}(x_k + 1) - 2y_k + 2c - 1 \quad by\ using...(2)$$

- Multiplying both side by (dx)

  $dx(d_1-d_2) = 2dy(x_k+1)-2dx(y_k)+2dx(c)-dx$
- Now we need to find decision parameter P_K

  $P_K = dx(d_1-d_2)$ and,
- $C = 2dy+2dx(c)-dx$ **which is constant**
- So new equation is.

  $P_K = 2dy(x_k)-2dx(y_k) + C$ ....(4)
- Now our next parameter will be

  $P_{K+1} = 2dy(x_{k+1})-2dx(y_{k+1}) + C$ .....(5)
- Subtracting P_k from PK_{+1}

  $P_{k+1}-P_k = 2dy(x_{k+1}-x_k)-2dx(y_{k+1}-y_k) + C-C$

Derivation

Note: here x is always incrementing so we can write x_{k+1} as $x_k + 1$

$P_{k+1} - P_k = 2dy(x_k - x_k + 1) - 2dx(y_{k+1} - y_k)$

$P_{k+1} = P_k + 2dy - 2dx(y_{k+1} - y_k)$........(6)

- when $P_k < 0$ then $(d_1 - d_2) < 0$
- So $d_1 < d_2$ then we will write y_{k+1} as y_k because current pixel's distance from intersection point y is smaller.so, we will have to choose current pixel. Then our formula will be:
- $P_{k+1} = P_k + 2dy - 2dx(y_k - y_k)$
- $P_{k+1} = P_k + 2dy$

Derivation

- And when $P_k > 0$ then $(d_1 - d_2) > 0$
- So $d_1 > d_2$ then we will write y_{k+1} as $y_k + 1$ because current pixel's distance from intersection point y is larger.so, we will have to choose upper pixel.
- At that time our formula will be:

  $P_{k+1} = P_k + 2dy - 2dx(y_k + 1 - y_k)$

  $P_{k+1} = P_k + 2dy - 2dx$
- We can say that $(y_{k+1} - y_k)$ value can either be 0 or 1. For Initial decision parameter
- From 4^{th} equation

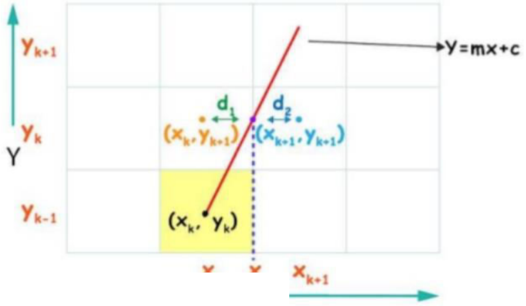
  $P_K = 2dy(x_k) - 2dx(y_k) + C$

  $P_K = 2dy(x_k) - 2dx(y_k) + 2dx(c) - dx + 2dy$

Derivation

- By using 1st equation

$$y_k = m(x_k) + c$$
$$c = y_k - m(x_k)$$
$$P_0 = 2dy(x_k) - 2dx(y_k) + 2dx(y_k - m(x_k) - dx \text{ (By using 2)}$$
$$= 2dy(x_k) - 2dx(y_k) + 2dxy_k - 2dyx_k + 2dy - dx$$
$$P_0 = 2dy - dx$$



- But if the slope of line is greater then 1 (m>1).
  then our Y coordinate will always be incremented and we have to choose between x_k or x_{k+1}.
  So, our Line equation will be:

$$Y_{k+1} = m(x) + c$$