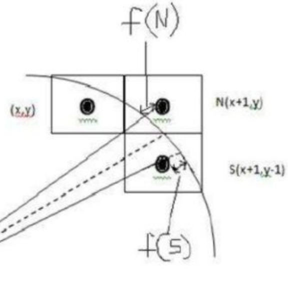
Derivation

Let $d_i = f(N) + f(S)$, where d can be called as "decision parameter", so that



if (d_i<=0),

then, N(x+1,y) is to be chosen as next pixel
i.e. $x_{i+1} = x_i+1$ and $y_{i+1} = y_i$,

and if (d_i>0),

then, S(x+1,y-1) is to be chosen as next pixel
i.e. $x_{i+1} = x_i+1$ and $y_{i+1} = y_i-1$.

Derivation

We know that for a circle,

$$x^2 + y^2 = r^2,$$

where r represents the radius of the circle, an input to the algorithm.

Errors can be represented as

$$f(N) = (xi + 1)^2 + yi^2 - r^2, \qquad -(1)$$

$$f(S) = (xi + 1)^2 + (yi - 1)^2 - r^2 \qquad -(2)$$

As $di = f(N) + f(S)$,

$$di = 2(xi+1)^2 + yi^2 + (yi-1)^2 - 2r^2 \qquad -(3)$$

Derivation

Calculating next decision parameter,

$_d_{i+1} = 2(x_i+2)^2 + y_{i+1}{}^2 + (y_{i+1}-1)^2 – 2r^2$ -(4)

from (4)- (3), we get,

$d_{i+1} – d_i = 2((x_i+2)^2-(x_i+1)^2) + (y_{i+1}{}^2 – y_i^2) + ((y_{i+1}-1)^2 + (y_i-1)^2)$

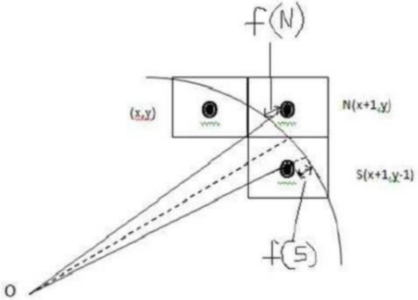
$_d_{i+1} = d_i + 2((x_i+2+x_i+1)(x_i+2-x_i-1)) + ((y_{i+1}+y_i)(y_{i+1}-y_i)) + ((y_{i+1}-1+y_i-1)(y_{i+1}-1-y_i+1))$

$_d_{i+1} = d_i + 2(2x_i+3) + ((y_{i+1}+y_i)(y_{i+1}-y_i)) + ((y_{i+1}-1+y_i-1)(y_{i+1}-1-y_i+1))$

Derivation

Now, if $(d_i<=0)$,

$x_{i+1}=x_i+1$ and $y_{i+1}=y_i$



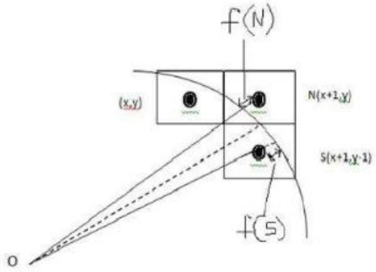
so that $d_{i+1} = d_i + 2(2x_i + 3) + ((y_{i+1}+y_i)(y_i-y_i)) + ((y_i-1+y_i-1)(y_i-1-y_i+1))$

$d_{i+1} = d_i + 2(2x_i + 3) + ((y_{i+1}+y_i)(0)) + ((y_i-1+y_i-1)(0))$

$$d_{i+1} = d_i + 4x_i + 6$$

Derivation

Else $(d_i > 0)$



$$d_{i+1} = d_i + 2(2x_i+3) + ((y_i-1+y_i)(y_i-1-y_i)) + ((y_i-2+y_i-1)(y_i-2-y_i+1))$$

$$d_{i+1} = d_i + 4x_i+6 + ((2y_i-1)(-1)) + ((2y_i-3)(-1))$$

$$d_{i+1} = d_i + 4x_i+6 - 2y_i - 2y_i + 1 + 3$$

$$d_{i+1} = d_i + 4(x_i - y_i) + 10$$