Continuity between curve segments

- If the tangent vectors of two cubic curve segments are equal at the join point, the curve has first-degree continuity, and is said to be ***C^1 continuous***
- If the direction and magnitude of *d / dt [Q(t)]* through the *n*th derivative are equal at the join point, the curve is called ***C^n continuous***
- If the directions (but not necessarily the magnitudes) of two segments' tangent vectors are equal at the join point, the curve has ***G^1 continuity***

Parametric Cubic Curves

- In order to assure *C2* continuity our functions must be of at least degree 3.
- Cubic has 4 degrees of freedom and can control 4 things.
- Use polynomials: x(*t*) of degree *n* is a function of *t*.

$$x(t) = \sum_{i=0}^{n} a_i t^n$$

- y(*t*) and z(*t*) are similar and each is handled independently

Lecture No 26 Topic: Hermite curves

- A cubic polynomial
- Polynomial can be specified by the position of, and gradient at, each endpoint of curve.
- Determine: $x = X(t)$ in terms of x_0, x_0', x_1, x_1'

Now: $X(t) = a_3t^3 + a_2t^2 + a_1t + a_0$

and $X^/(t) = 3a_3t^2 + 2a_2t + a_1$

Hermite Specification

Finding Hermit coefficients

Substituting for t at each endpoint:

$x_0 = X(0) = a_0$ $\quad$ $x_0' = X'(0) = a_1$

$x_1 = X(1) = a_3 + a_2 + a_1 + a_0$ $\quad$ $x_1' = X'(1) = 3a_3 + 2a_2 + a_1$

And the solution is:

$a_0 = x_0$ $\quad$ $a_1 = x_0'$

$a_2 = -3x_0 - 2x_0' + 3x_1 - x_1'$ $\quad$ $a_3 = 2x_0 + x_0' - 2x_1 + x_1'$

The Hermite matrix: M_H

The resultant polynomial can be expressed in matrix form:

$X(t) = t^T M_H q$ (q is the control vector)

$$X(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -2 & 1 \\ -3 & -2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_0' \\ x_1 \\ x_1' \end{bmatrix}$$

We can now define a parametric polynomial for each coordinate required independently, ie. X(t), Y(t) and Z(t)