Hermite Basis (Blending) Functions

$$X(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -2 & 1 \\ -3 & -2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_0' \\ x_1 \\ x_1' \end{bmatrix}$$

$$= \underline{\underline{(2t^3 - 3t^2 + 1)}}x_0 + \underline{\underline{(t^3 - 2t^2 + t)}}x_0' + \underline{\underline{(-2t^3 + 3t^2)}}x_1 + \underline{\underline{(t^3 - t^2)}}x_1'$$

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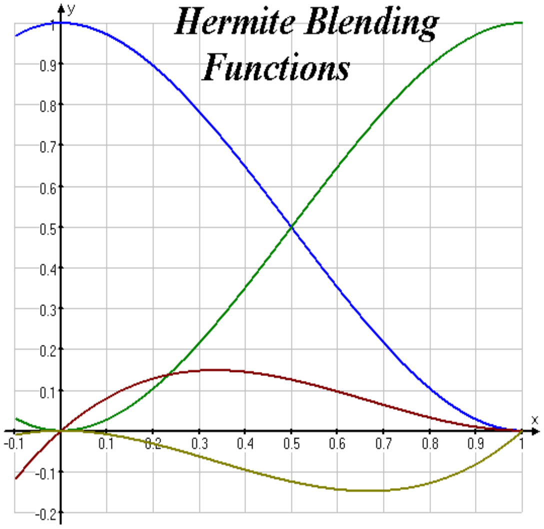
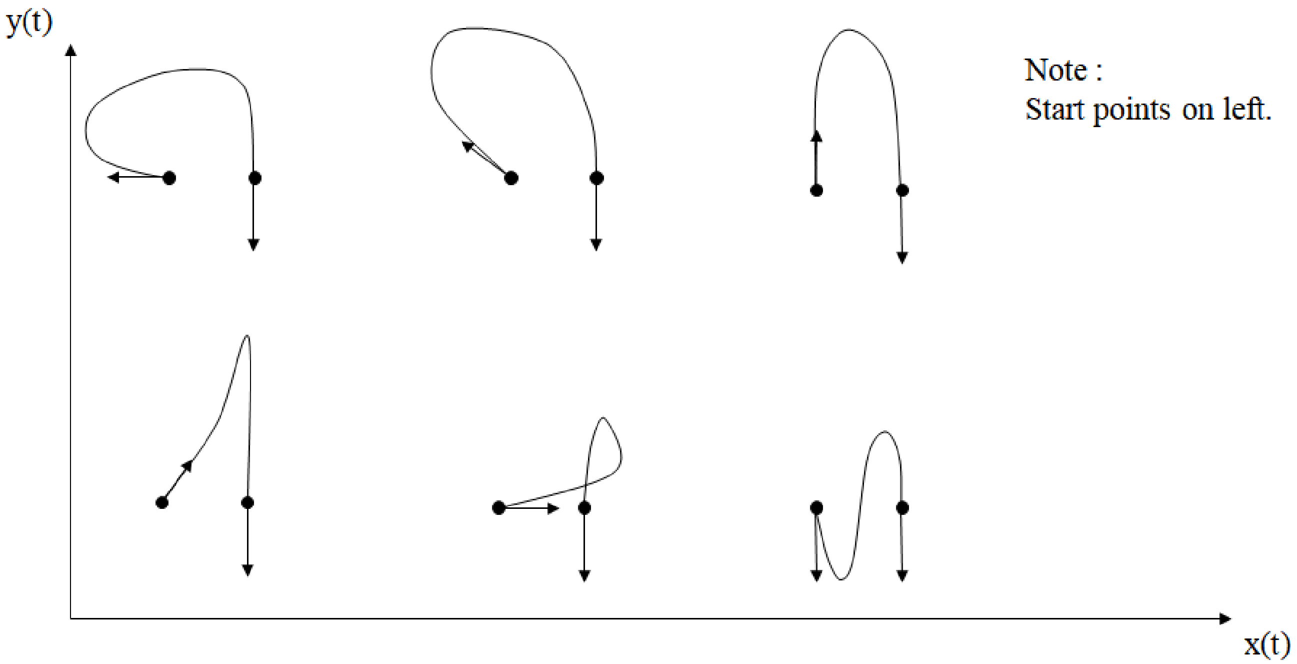
The graph shows the shape of the four basis functions – often called *blending functions.*

They are labelled with the elements of the control vector that they weight.

Note that at each end only position is non-zero, so the curve must touch the endpoints



Family of Hermite curves.



Lecture No 27 Topic: Bézier Curves

- Hermite cubic curves are difficult to model – need to specify point and gradient.
- More intuitive to only specify points.
- Pierre Bézier (an engineer at Renault) specified 2 endpoints and 2 additional control points to specify the gradient at the endpoints.
- Can be derived from Hermite matrix:
  - Two end control points specify tangent

Bézier Curves

Note the Convex Hull has been shown as a dashed line – used as a bounding extent for intersection purposes.