Some simple equations can be solved

Some simple equations can be solved analytically:

$$x^2 + 4x + 3 = 0$$

$$\text{Analytic solution } roots = \frac{-4 \pm \sqrt{4^2 - 4(1)(3)}}{2(1)}$$

$$x = -1 \; and \; x = -3$$

Many other equations have no analytical solution:

$$\left.\begin{matrix} x^9 - 2x^2 + 5 = 0 \\ x = e^{-x} \end{matrix}\right\} \text{No analytic solution}$$

Lecture No 41 Topic: Methods for Solving Nonlinear Equation

- Bisection Method
- Newton-Raphson Method
- Secant Method

Solution of Systems of Linear Equations

$$x_1 + x_2 = 3$$

$$x_1 + 2x_2 = 5$$

We can solve it as :

$$x_1 = 3 - x_2, \qquad 3 - x_2 + 2x_2 = 5$$

$$\Rightarrow x_2 = 2,\ x_1 = 3 - 2 = 1$$

What to do if we have

1000 equations in 1000 unknowns.

Cramer’s Rule is Not Practical

Cramer's Rule can be used to solve the system:

$$x_1 = \frac{\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}} = 1, \quad x_2 = \frac{\begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}} = 2$$

But Cramer's Rule is not practical for large problems.

To solve N equations with N unknowns, we need $(N+1)(N-1)N!$ multiplications.

To solve a 30 by 30 system, 2.3×10^{35} multiplications are needed.

A super computer needs more than 10^{20} years to compute this.

Lecture No44Topic: Methods for Solving Systems of Linear Equations

- Naive Gaussian Elimination
- Gaussian Elimination with Scaled Partial Pivoting
- Algorithm for Tri-diagonal Equations