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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

BME504:Heat and Mass Transfer

Lecture 5

Instructor:
Aditya Veer Gautam

HEAT GENERATION

- A medium through which heat is conducted may involve the conversion of electrical, nuclear, or chemical energy into heat (or thermal) energy. In heat conduction analysis, such conversion processes are characterized as heat generation.
- The temperature of a resistance wire rises rapidly when electric current passes through it as a result of the electrical energy being converted to heat at a rate of I^2R
- Heat generation is a ***volumetric phenomenon***
- The total rate of heat generation in a medium of ***volume V***

$$\dot{G} = \int_V \dot{g} dV$$

- In the special case of uniform heat generation, as in the case of electric resistance heating throughout a homogeneous material, the equation reduces to G = $\dot{g}$V, where $\dot{g}$ is the constant rate of heat generation per unit volume.

ONE-DIMENSIONAL GENERAL DIFFERENTIAL HEAT CONDUCTION EQUATION IN THE RECTANGULAR COORDINATE SYSTEM

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho CA\Delta x(T_{t+\Delta t} - T_t)$$

$$\dot{G}_{\text{element}} = \dot{g}V_{\text{element}} = \dot{g}A\Delta x$$

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{g}A\Delta x = \rho CA\Delta x\,\frac{T_{t+\Delta t} - T_t}{\Delta t}$$

$$-\frac{1}{A}\,\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} + \dot{g} = \rho C\,\frac{T_{t+\Delta t} - T_t}{\Delta t}$$

$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \rho C \frac{\partial T}{\partial t}$

A

- Taking the limit as Δx→0 and Δt→0 yields

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

Variable thermal conductivity	Constant thermal conductivity
$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$

Steady-state: $(\partial/\partial t = 0)$ $$\frac{d^2 T}{dx^2} + \frac{\dot{g}}{k} = 0$$

Transient, no heat generation: $(\dot{g} = 0)$ $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Steady-state, no heat generation: $(\partial/\partial t = 0 \text{ and } \dot{g} = 0)$ $$\frac{d^2 T}{dx^2} = 0$$

ONE-DIMENSIONAL GENERAL DIFFERENTIAL HEAT CONDUCTION EQUATION IN THE CYLINDRICAL COORDINATE SYSTEM

- Variable thermal conductivity

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

- Constant thermal conductivity

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Steady-state: $(\partial/\partial t = 0)$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$
Transient, no heat generation: $(\dot{g} = 0)$	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$
Steady-state, no heat generation: $(\partial/\partial t = 0$ and $\dot{g} = 0)$	$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$

ONE-DIMENSIONAL GENERAL DIFFERENTIAL HEAT CONDUCTION EQUATION IN THE SPHERICAL COORDINATE SYSTEM

- Variable thermal conductivity

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial T}{\partial r}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

- Constant thermal conductivity

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Steady-state: $(\partial/\partial t = 0)$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

Transient, no heat generation: $(\dot{g} = 0)$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

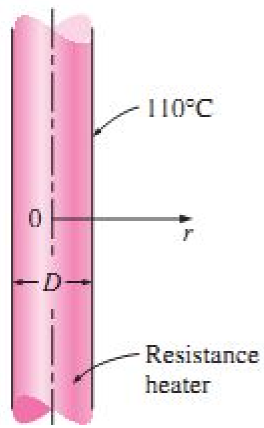
Steady-state, no heat generation: $(\partial/\partial t = 0 \text{ and } \dot{g} = 0)$

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0 \quad \text{or} \quad r\frac{d^2T}{dr^2} + 2\frac{dT}{dr} = 0$$

Questions

Q1 An iron is left unattended and its base temperature rises as a result of resistance heating inside. When will the rate of heat generation inside the iron be equal to the rate of heat loss from the iron?

Q2 A 2-kW resistance heater wire with thermal conductivity of k = 20 W/m · °C, a diameter of D = 5 mm, and a length of L = 0.7 m is used to boil water. If the outer surface temperature of the resistance wire is T_s =110°C, determine the temperature at the center of the wire.



Questions

Q3 Consider a long solid cylinder of radius r_0 = 4 cm and thermal conductivity k = 25 W/m · °C. Heat is generated in the cylinder uniformly at a rate of g_0 = 35 W/cm3. The side surface of the cylinder is maintained at a constant temperature of T_s =80°C. The variation of temperature in the cylinder is given by

$$T(r) = \frac{\dot{g} r_0^2}{k}\left[1 - \left(\frac{r}{r_0}\right)^2\right] + T_s$$

Based on this relation, determine (a) if the heat conduction is steady or transient, (b) if it is one-, two-, or three-dimensional, and (c) the value of heat flux on the side surface of the cylinder at r = r_0.

Questions

Q4 In a nuclear reactor, 1-cm-diameter cylindrical uranium rods cooled by water from outside serve as the fuel. Heat is generated uniformly in the rods (k =29.5 W/m · °C) at a rate of 7 X 10^7 W/m3. If the outer surface temperature of rods is 175°C, determine the temperature at their center

Q5 Consider a large 3-cm-thick stainless steel plate (k 15.1 W/m · °C) in which heat is generated uniformly at a rate of 5 x10^5 W/m3. Both sides of the plate are exposed to an environment at 30°C with a heat transfer coefficient of 60 W/m2 · °C. Explain where in the plate the highest and the lowest temperatures will occur, and determine their values