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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

BME504:Heat and Mass Transfer

Lecture 6

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GENERAL HEAT CONDUCTION EQUATION

- Fourier-Biot Equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(1) *Steady-state:*
(called the **Poisson equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = 0$$

(2) *Transient, no heat generation:*
(called the **diffusion equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:*
(called the **Laplace equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

BOUNDARY AND INITIAL CONDITIONS

- The differential equations do not incorporate any information related to the conditions on the surfaces such as the surface temperature or a specified heat flux.
- The mathematical expressions of the thermal conditions at the boundaries are called the boundary conditions.
- From a mathematical point of view, solving a differential equation is essentially a process of removing derivatives, or an integration process, and thus the solution of a differential equation typically involves arbitrary constants. In all the above equations we have 2nd order differential equations so we need at least 2 boundary condition to solve then completely (to get unique solution)

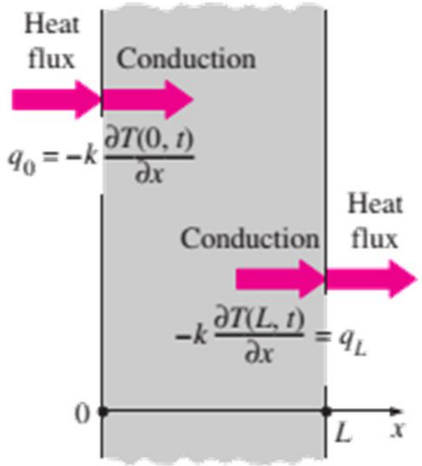
BOUNDARY AND INITIAL CONDITIONS

- **Initial Condition** The temperature at any point on the body at a specified time also depends on the condition of the wall at the beginning of the heat conduction process. Such a condition, which is usually specified at time t=0, is called the initial condition, which is a mathematical expression for the temperature distribution of the medium initially
- We need only one initial condition for a heat conduction problem regardless of the dimension since the conduction equation is first order in time.
- **Boundary Conditions** There are approximately 5 kinds of boundary conditions
  - ***Specified Temperature Boundary Condition*:** For one-dimensional heat transfer through a plane wall of thickness L

$$T(0, t) = T_1$$
$$T(L, t) = T_2$$

Specified Heat Flux Boundary Condition

Specified Heat Flux Boundary Condition



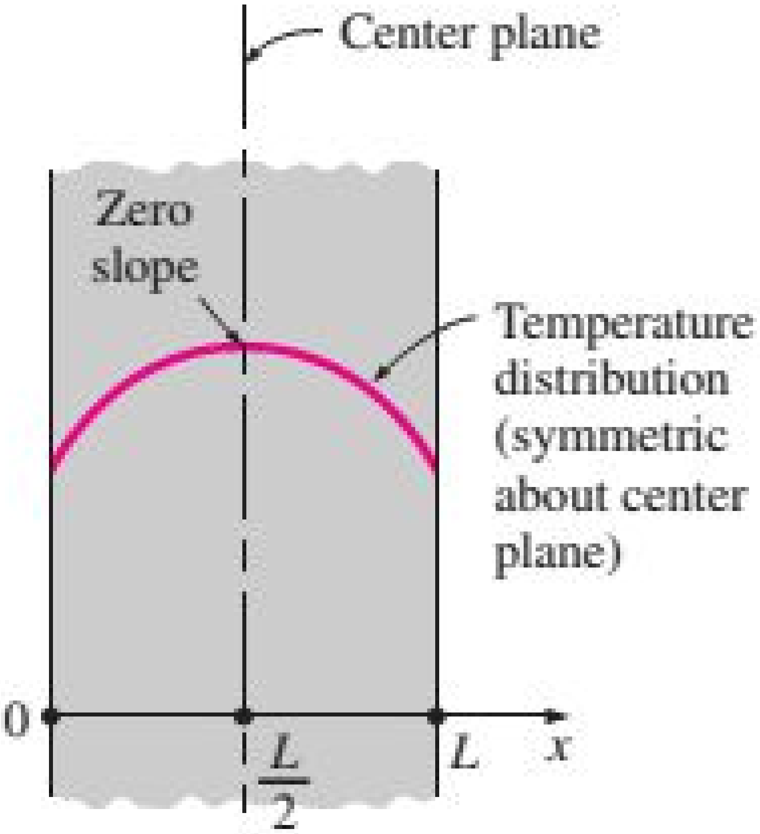
The sign of the specified heat flux is determined by inspection: positive if the heat flux is in the positive direction of the coordinate axis, and negative if it is in the opposite direction

- On an insulated surface, the first derivative of temperature with respect to the space variable (the temperature gradient) in the direction normal to the insulated surface is zero. This also means that the temperature function must be perpendicular to an insulated surface since the slope of temperature at the surface must be zero.

$$\frac{\partial T(0,t)}{\partial x} = 0$$

Specified Heat Flux Boundary Condition

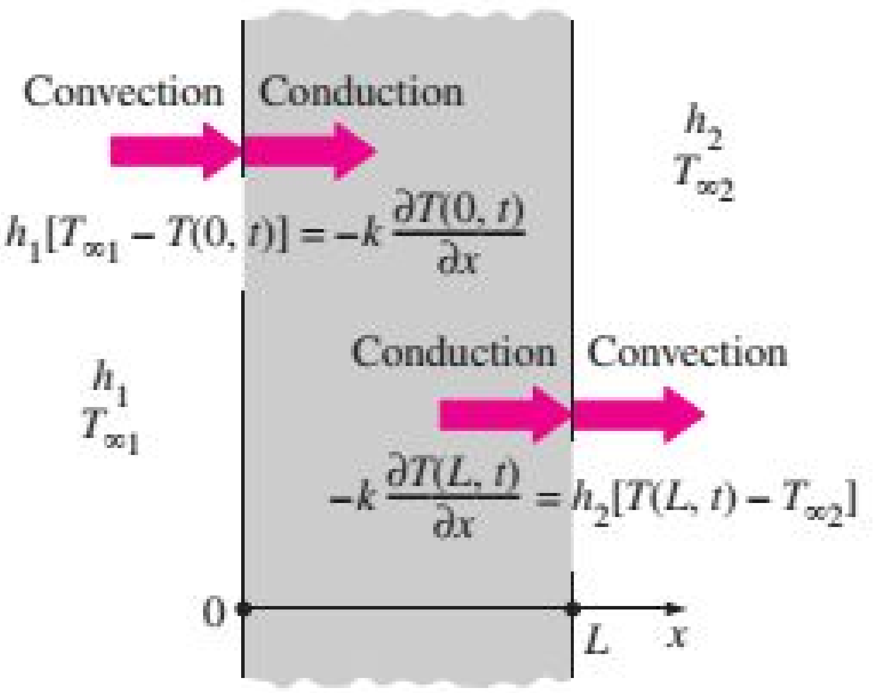
Thermal Symmetry:



$$\frac{\partial T(L/2, t)}{\partial x} = 0$$

Specified Heat Flux Boundary Condition

Convection Boundary Condition:



$$-k \frac{\partial T(0, t)}{\partial x} = h_1[T_{\infty 1} - T(0, t)]$$

$$-k \frac{\partial T(L, t)}{\partial x} = h_2[\mathrm{T}(L, t) - T_{\infty 2}]$$

Questions

Q1 Consider a large plane wall of thickness L = 0.4 m, thermal conductivity k = 2.3 W/m · °C, and surface area A = 20 m2. The left side of the wall is maintained at a constant temperature of T1 = 80°C while the right side loses heat by convection to the surrounding air at T = 15°C with a heat transfer coefficient of h = 24 W/m2 · °C. Assuming constant thermal conductivity and no heat generation in the wall, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (b) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (c) evaluate the rate of heat transfer through the wall.

Q2 Consider a solid cylindrical rod of length 0.15 m and diameter 0.05 m. The top and bottom surfaces of the rod are maintained at constant temperatures of 20°C and 95°C, respectively, while the side surface is perfectly insulated. Deter-mine the rate of heat transfer through the rod if it is made of (a) copper, k = 380 W/m · °C, (b) steel, k = 18 W/m · °C, and (c) granite, k = 1.2 W/m · °C.

Questions

Q3 A spherical container of inner radius $r1 = 2$ m, outer radius $r2 = 2.1$ m, and thermal conductivity $k = 30$ W/m · °C is filled with iced water at 0°C. The container is gaining heat by convection from the surrounding air at $T = 25$°C with a heat transfer coefficient of $h = 18$ W/m2 · °C. Assuming the inner surface temperature of the container to be 0°C, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the container, (b) obtain a relation for the variation of temperature in the container by solving the differential equation, and (c) evaluate the rate of heat gain to the iced water

Q4 A container consists of two spherical layers, A and B, that are in perfect contact. If the radius of the interface is r_0, express the boundary conditions at the interface

Questions

Q5 Consider a large plane wall of thickness L 0.3 m, thermal conductivity k 2.5 W/m · °C, and surface area A 12 m2. The left side of the wall at x 0 is subjected to a net heat flux of q·0 700 W/m2 while the temperature at that sur-face is measured to be T1 80°C. Assuming constant thermal conductivity and no heat generation in the wall, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (b) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (c) evaluate the temperature of the right surface of the wall at x L.