- SHEAR STRESS AND SHEAR STRAIN
- When the external force acting on a component tends to slide the adjacent planes with respect to each other, the resulting stresses on these planes are called direct shear stresses. Two plates held together by means of a rivet are shown in Fig. 4.3 (a). The average shear stress in the rivet is given by

$$\tau = \frac{P}{A}$$

- where, t = shear stress (N/mm2 or MPa), A = cross-sectional area of the rivet (mm2

A plane rectangular element, cut from the component and subjected to shear force, is shown in Fig. 4.4(a). Shear stresses cause a distortion in the original right angles. The shear strain (g) is defi ned as the change in the right angle of a shear element. Within the elastic limit, the stress–strain relationship is given by

$$\tau = G\gamma$$

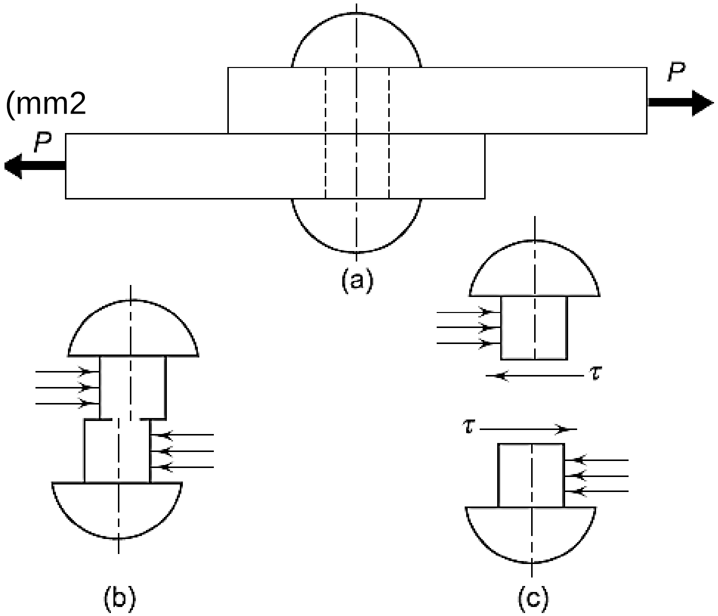


Fig. 4.3 *(a) Riveted Joint (b) Shear Deformation (c) Shear Stress*

where,

γ = shear strain (radians)

G is the constant of proportionality known as *shear modulus* or *modulus of rigidity* (in N/mm^2 or MPa).

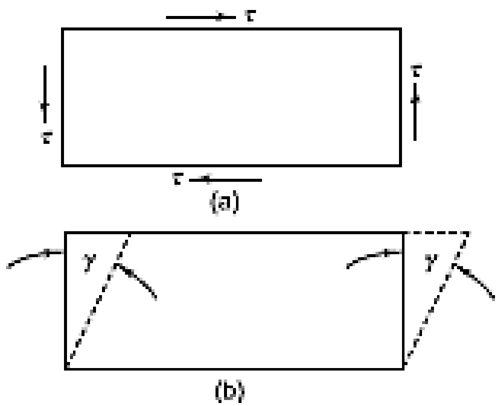


Fig. 4.4 *(a) Element Loaded in Pure Shear (b) Shear Strain*

For carbon steels, $G = 80\,000$ N/mm^2

For grey cast iron, $G = 40\,000$ N/mm^2

The relationship between the modulus of elasticity, the modulus of rigidity and the Poisson's ratio is given by,

$$E = 2G\,(1 + \mu) \qquad (4.10)$$

where μ is Poisson's ratio. *Poisson's ratio* is the ratio of strain in the lateral direction to that in the axial direction.

For carbon steels, $\mu = 0.29$

For grey cast iron, $\mu = 0.21$

The permissible shear stress is given by,

$$\tau = \frac{S_{sy}}{(fs)} \qquad (4.11)$$

where,

S_{sy} = yield strength in shear (N/mm^2 or MPa)

It will be proved at a later stage that the yield strength in shear is 50% of the yield strength in tension, according to the principal shear stress theory of failure.

STRESSES DUE TO BENDING MOMENT

A straight beam subjected to a bending moment Mb is shown in Fig. 4.5(a). The beam is subjected to a combination of tensile stress on one side of the neutral axis and compressive stress on the other. Such a stress distribution can be visualized by bending a thick leather belt. Cracks will appear on the outer surface, while folds will appear on the inside. Therefore, the outside fi bres are in tension, while the inside fi bres are in compression. The bending stress at any fi bre is given by,

$$\sigma_b = \frac{M_b\, y}{I} \qquad (4.12)$$

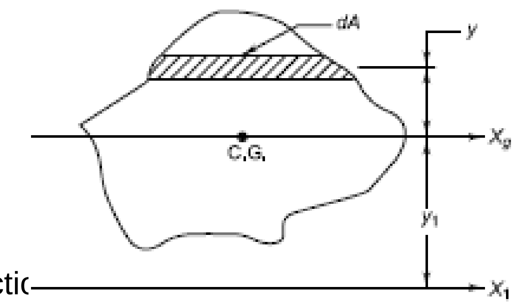
where,

σ_b = bending stress at a distance of y from the neutral axis (N/mm^2 or MPa)

M_b = applied bending moment (N-mm)

I = moment of inertia of the cross-section about the neutral axis (mm^4)

- The bending stress is maximum in a fi bre, which is farthest from the neutral axis. The distribution of stresses is linear and the stress is proportional to the distance from the neutral axis.
- (i) The beam is straight with uniform crosssection.
- (ii) The forces acting on the beam lie in a plane perpendicular to the axis of the beam.
- (iii) The material is homogeneous, isotropic and obeys Hooke's law.
- (iv) Plane cross-sections remain plane after bending.
- The moment of inertia in Eq. (4.12) is the area moment of inertia. For a rectangular cross-sectic



$$I = \frac{bd^3}{12} \qquad (4.13)$$

where,

b = distance parallel to the neutral axis (mm)

d = distance perpendicular to the neutral axis (mm)

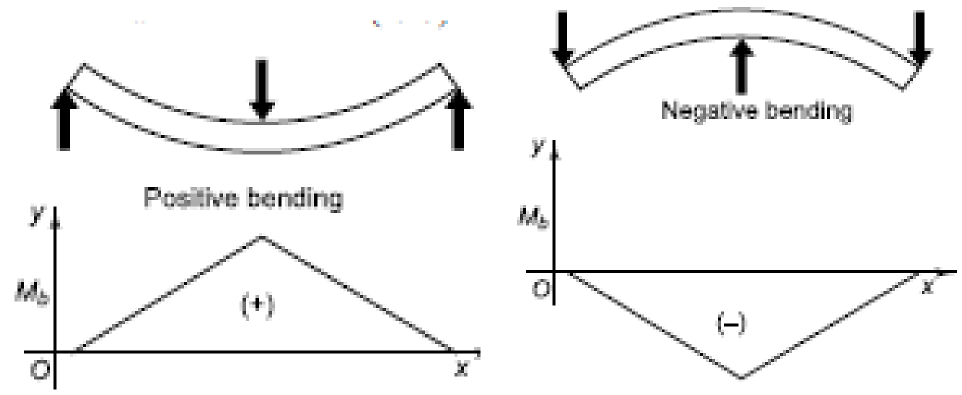
For a circular cross-section,

$$I = \frac{\pi d^4}{64} \qquad (4.14)$$

where d is the diameter of the cross-section.

When the cross-section is irregular, as shown in Fig. 4.6, the moment of inertia about the centroidal axis X_g is given by,

$$I_{xg} = \int y^2\, dA \qquad (4.15)$$



The parallel-axis theorem for this area is given by the expression,

$$I_{x1} = I_{xg} + Ay_1^2 \qquad (4.16)$$

where,

I_{x1} = moment of inertia of the area about X_1 axis, which is parallel to the axis X_g, and located at a distance y_1 from X_g

I_{xg} = moment of inertia of the area about its own centroidal axis

A = area of the cross-section.

- **STRESSES DUE TO TORSIONAL MOMENT**

- A transmission shaft, subjected to an external torque, is shown in Fig. 4.8 (a). The internal stresses, which are induced to resist the action of twist, are called torsional shear stresses. The torsional shear stress is given by

$$\tau = \frac{M_t\, r}{J} \qquad (4.17)$$

where,

τ = torsional shear stress at the fibre (N/mm^2 or MPa)

M_t = applied torque (N-mm)

r = radial distance of the fibre from the axis of rotation (mm)

J = polar moment of inertia of the cross-section about the axis of rotation (mm^4)

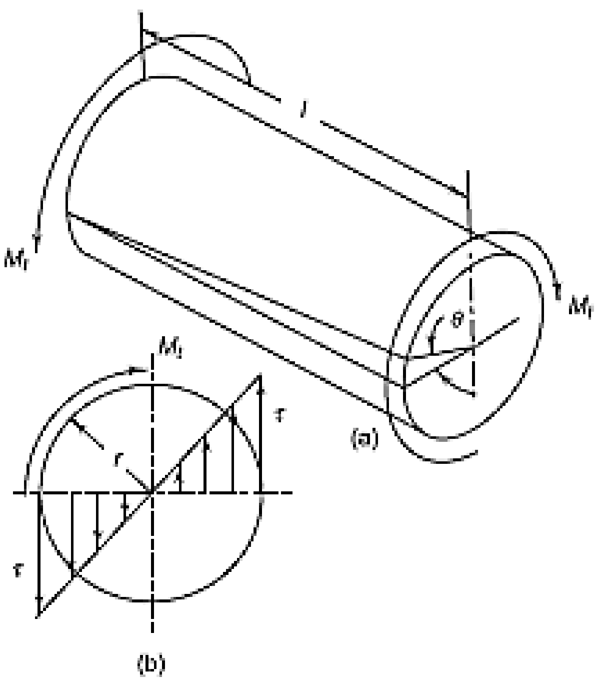


Fig. 4.8 *(a) Shaft Subjected to Torsional Moment (b) Distribution of Torsional Shear Stresses*

The distribution of torsional shear stresses is shown in Fig. 4.8 (b). The stress is maximum at the outer fibre and zero at the axis of rotation. The angle of twist is given by

$$\theta = \frac{M_t\, l}{JG} \qquad (4.18)$$

where,

θ = angle of twist (radians)

l = length of the shaft (mm)

Equations (4.17) and (4.18) are based on the following assumptions:

(i) The shaft is straight with a circular cross-section.

(ii) A plane transverse section remains plane after twisting.

(iii) The material is homogeneous, isotropic and obeys Hooke's law.

The polar moment of inertia of a solid circular shaft of diameter d is given by

$$J = \frac{\pi d^4}{32} \qquad (4.19)$$

For a hollow circular cross-section,

$$J = \frac{\pi\,(d_o^4 - d_i^4)}{32} \qquad (4.20)$$

Substituting Eqs (4.19) and (4.20) in Eq. (4.18) and converting θ from radians to degrees,

$$\theta = \frac{584\, M_t\, l}{G d^4} \quad \text{(deg)} \qquad (4.21)$$

$$\theta = \frac{584\, M_t\, l}{G\,(d_o^4 - d_i^4)} \quad \text{(deg)} \qquad (4.22)$$

In many problems of machine design, it is required to calculate torque from the power transmitted and the speed of rotation. This relationship is given by,

$$kW = \frac{2\pi n M_t}{60 \times 10^6}$$

where,

kW = transmitted power (kW)

Mt = torque (N-mm)

n = speed of rotation (rpm)

- ECCENTRIC AXIAL LOADING
- line of action of force passes through the centroid of the cross-section.
- There are certain mechanical components subjected to an external force, tensile or compressive, which does not pass through the centroid of the cross-section. A typical example of such an eccentric loading is shown in Fig. 4.9(a).
- According to the principle of statics, the eccentric force P can be replaced by a parallel force P passing through the centroidal axis along with a couple (P X e) as shown in Figs 4.9(b) and (c) respectively.
- In Fig. 4.9(b), the force P causes a uniformly distributed tensile stress of magnitude (P/A). In Fig. 4.9(c),
- the couple causes bending stress of magnitude (Pey/I). The resultant stresses at the cross-section are obtained by the principle of superimposition of stresses. They are given by,

$$\sigma = \frac{P}{A} \pm \frac{Pey}{I}$$

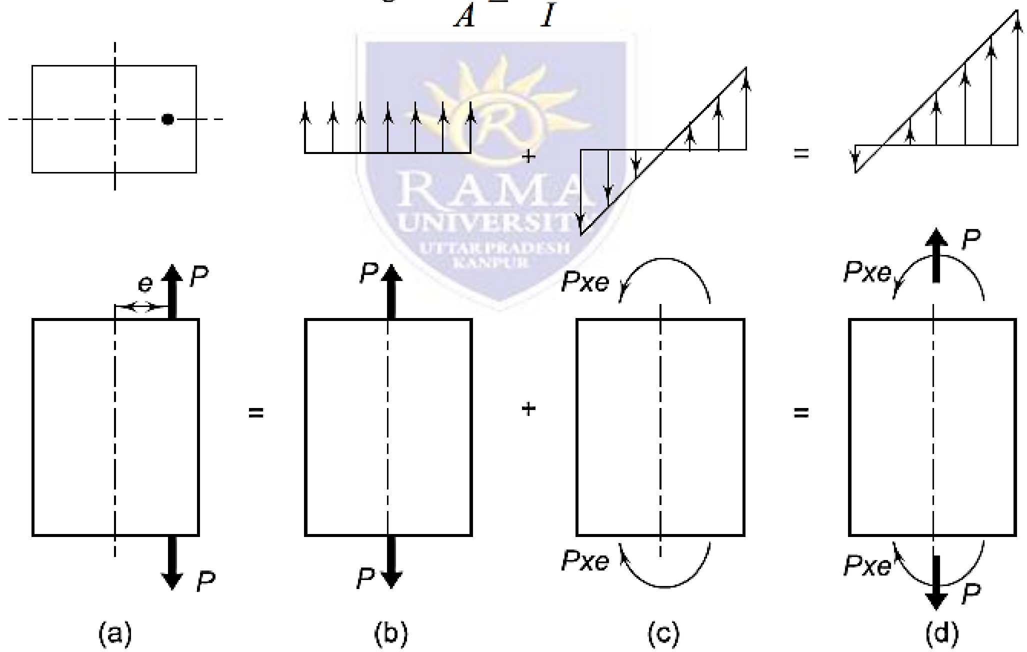


Fig. 4.9 *Eccentric Axial Load*