From the figure,

$$\sigma_1 = -\sigma_2 = \tau_{xy} \quad \text{and} \quad \sigma_3 = 0$$

Substituting these values in Eq. (4.42),

$$S_{yt} = \sqrt{3}\,\tau_{xy}$$

Replacing (τ_{xy}) by S_{sy},

$$S_{sy} = \frac{S_{yt}}{\sqrt{3}} = 0.577\,S_{yt} \qquad (4.45)$$

Therefore, according to the distortion-energy theory, the yield strength in shear is 0.577 times the yield strength in tension.

Region of Safety The construction of the region of safety is illustrated in Fig. 4.36. The two principal stresses σ_1 and σ_2 are plotted on the X and Y axes respectively. Tensile stresses are considered as positive, while compressive stresses as negative.

It should be noted that,

$$x^2 - xy + y^2 = a^2$$

is an equation of an ellipse whose semi-major axis is $(\sqrt{2}a)$ and semi-minor axis is $(\sqrt{2/3}\,a)$.

For bi-axial stresses,

$$\sigma_3 = 0$$

Substituting this value in Eq. (4.42),

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = S_{yt}^2$$

The above equation indicates an ellipse whose semi-major axis is $(\sqrt{2}\,S_{yt})$ and semi-minor axis is $\left[\sqrt{\frac{2}{3}}S_{yt}\right]$.

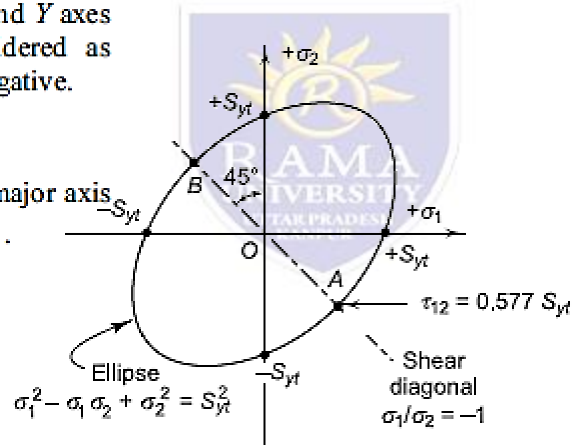

Fig. 4.36 *Boundary for Distortion Energy Theory under Bi-axial Stresses*

Shear Diagonal As mentioned in the previous section, shear diagonal or line of pure shear is the locus of all points, corresponding to pure shear stress. The condition for the line of shear is,

$$\sigma_1 = -\sigma_2 = \tau_{12}$$

The above equation can be written as

$$\frac{\sigma_1}{\sigma_2} = -1 = -\tan(45°)$$

A line $\overline{AB}$ is constructed in such a way that it passes through the origin O and makes an angle of $-45°$ with the Y-axis. This line is called shear diagonal or line of pure shear. This line intersects the ellipse at two points A and B.

A is the point of intersection of the ellipse and the line AB. The coordinates of the point A are obtained by solving the following two equations simultaneously,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = S_{yt}^2$$

$$\sigma_1/\sigma_2 = -1$$

Solving two equations simultaneously,

$$\sigma_1 = -\sigma_2 = +\frac{1}{\sqrt{3}}S_{yt}$$

Since $\quad \sigma_1 = -\sigma_2 = \tau_{12}$

$$\tau_{12} = \frac{1}{\sqrt{3}}S_{yt} = 0.577S_{yt}$$

Since the point A is on the borderline, this is the limiting value for shear stress.

or $\quad S_{sy} = 0.577\,S_{yt}$

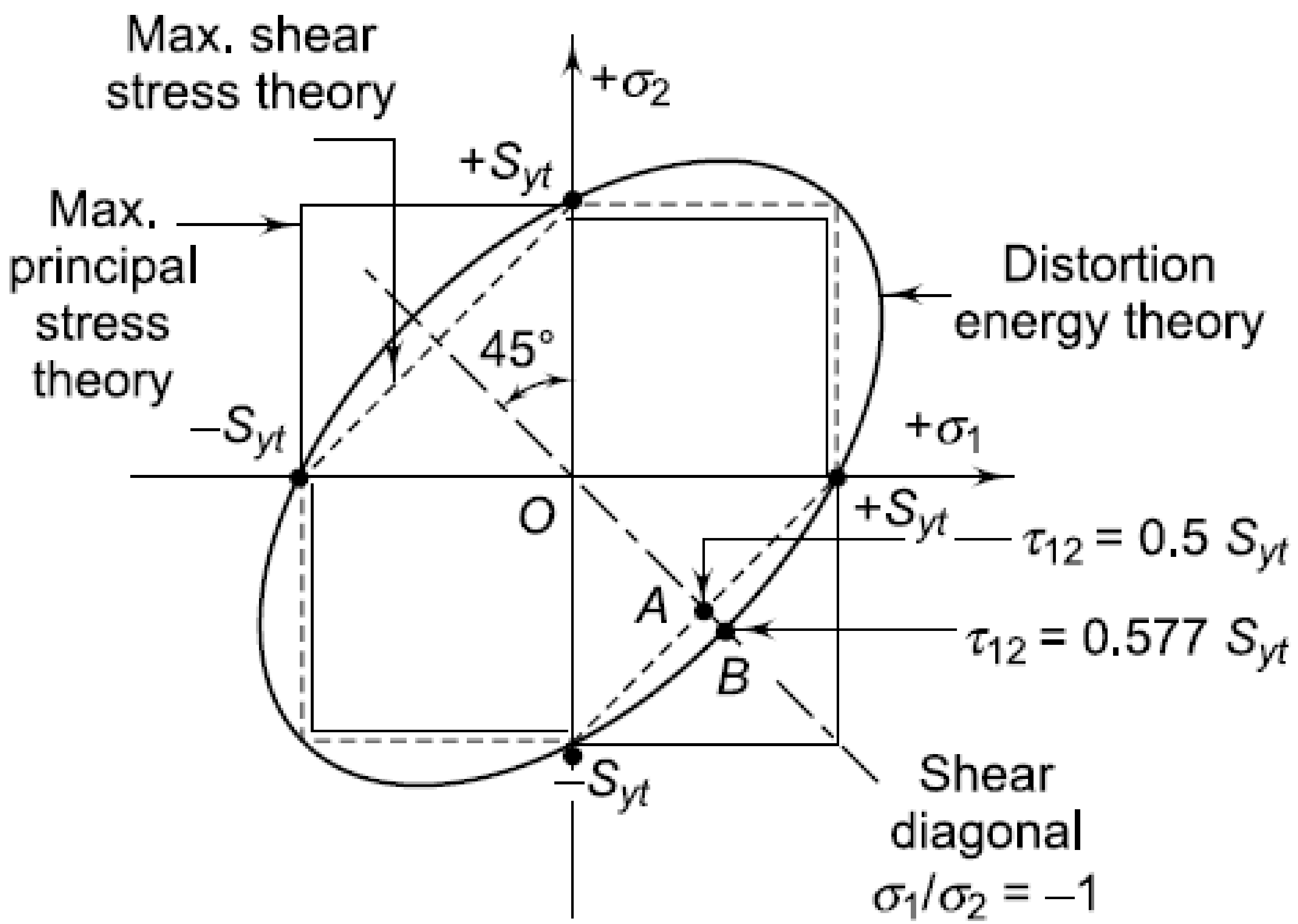

Fig. 4.37 *Comparison of Theories of Failure*

- SELECTION AND USE OF FAILURE THEORIES
- (i) Ductile materials typically have the same tensile strength and compressive strength. Also, yielding is the criterion of failure in ductile materials. In maximum shear stress theory and distortion energy theory, it is assumed that the yield strength in tension (Syt) is equal to the yield strength in compression (Syc). Also, the criterion of failure is yielding. Therefore, maximum shear stress theory and distortion energy theory are used for ductile materials.
- (ii) Distortion energy theory predicts yielding with precise accuracy in all four quadrants. The design calculations involved in this theory are slightly complicated as compared with those of maximum shear stress theory.
- (iii) The hexagonal diagram of maximum shear stress theory is inside the ellipse of distortion energy theory. Therefore, maximum shear stress theory gives results on the conservative side. On the other hand, distortion energy theory is slightly liberal.
- (iv) The graph of maximum principal stress theory is the same as that of maximum shear stress theory in the fi rst and third quadrants. However, the graph of maximum principal stress theory is outside the ellipse of distortion energy theory in the second and fourth quadrants. Thus, it would be dangerous to apply maximum principal stress theory in these regions, since it might predict safety, when in fact no safety exists.
- (v) Maximum shear stress theory is used for ductile materials, if dimensions need not be held too close and a generous factor of safety is used. The calculations involved in this theory are easier than those of distortion energy theory.
- (vi) Distortion energy theory is used when the factor of safety is to be held in close limits and the cause of failure of the component is being investigated. This theory predicts the failure most accurately.
- (vii) The compressive strength of brittle materials is much higher than their tensile strength. Therefore, the failure criterion should show a difference in tensile and compressive strength. On this account, maximum principal stress theory is used for brittle materials. Also, brittle materials do not yield and they fail by fracture.
- To summarise, the maximum principal stress theory is the proper choice for brittle materials. For ductile materials, the choice of theory depends on the level of accuracy required and the degree of computational diffi culty the designer is ready to face. For ductile materials, the most accurate way to design is to use distortion energy theory of failure and the easiest way to design is to apply maximum shear stress theory

- A cantilever beam of rectangular
- cross-section is used to support a pulley as shown
- in Fig. 4.38 (a). The tension in the wire rope is 5
- kN. The beam is made of cast iron FG 200 and the
- factor of safety is 2.5. The ratio of depth to width of
- the cross-section is 2. Determine the dimensions of
- the cross-section of the beam.

Solution

Given $P = 5$ kN $\quad S_{ut} = 200$ N/mm^2
$(fs) = 2.5 \quad d/w = 2$

Step I *Calculation of permissible bending stress*

$$\sigma_b = \frac{S_{ut}}{(fs)} = \frac{200}{2.5} = 80 \text{ N/mm}^2$$

Step II *Calculation of bending moments*

The forces acting on the beam are shown in Fig. 4.38(b). Referring to the figure,

$$(M_b)_{\text{at B}} = 5000 \times 500 = 2500 \times 10^3 \text{ N-mm}$$
$$(M_b)_{\text{at A}} = 5000 \times 500 + 5000 \times 1500$$
$$= 10000 \times 10^3 \text{ N-mm}$$

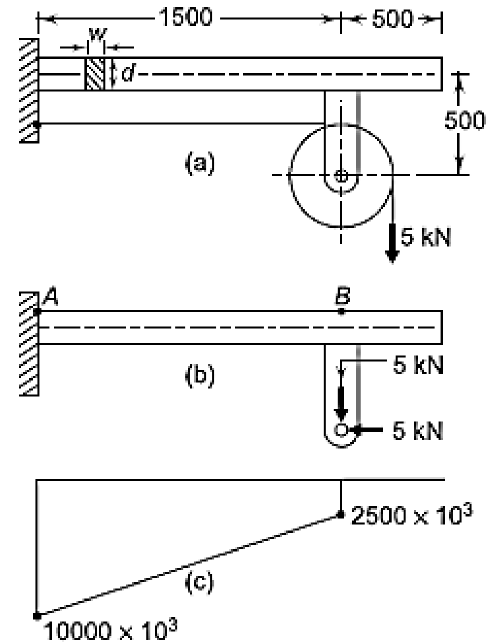

Fig. 4.38

tep III *Calculation of dimensions of cross-section*

he bending moment diagram is shown in Fig. 4.38(c). he cross-section at A is subjected to maximum bend- ig stress. For this cross-section,

$$y = \frac{d}{2} = w \qquad I = \frac{1}{12}[(w)(2w)^3] = \frac{2}{3}w^4 \text{ mm}^4$$

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad 80 = \frac{(10000 \times 10^3)(w)}{\left(\frac{2}{3}w^4\right)}$$

herefore,

$$w = 57.24 \text{ mm or } 60 \text{ mm} \qquad d = 2w = 120 \text{ mm}$$

- A wall bracket with a rectangular
- cross-section is shown in Fig. 4.39. The depth of
- the cross-section is twice of the width. The force
- P acting on the bracket at 600 to the vertical is 5
- kN. The material of the bracket is grey cast iron
- FG 200 and the factor of safety is 3.5. Determine
- the dimensions of the cross-section of the bracket.
- Assume maximum normal stress theory of failure.

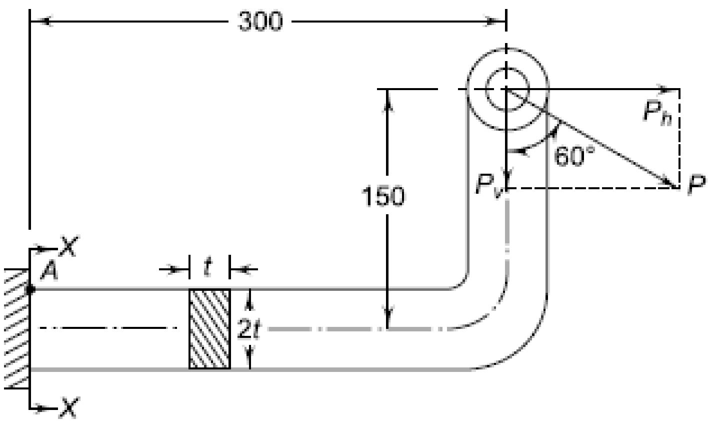

Fig. 4.39 *Wall Bracket*

Solution

Given $P = 5$ kN

$S_{ut} = 200$ N/ mm^2 $(fs) = 3.5$ $d/w = 2$

Step I *Calculation of permissible stress*

$$\sigma_{max} = \frac{S_{ut}}{(fs)} = \frac{200}{3.5} = 57.14 \text{ N/mm}^2 \qquad \text{(i)}$$

Step II *Calculation of direct and bending tensile stresses*

The stress is maximum at the point A in the section XX. The point is subjected to combined bending and direct tensile stresses. The force P is resolved into two components—horizontal component P_h and vertical component P_v.

$$P_h = P \sin 60° = 5000 \sin 60° = 4330.13 \text{ N}$$
$$P_v = P \cos 60° = 5000 \cos 60° = 2500 \text{ N}$$

The bending moment at the section XX is given by

$$M_b = P_h \times 150 + P_v \times 300$$
$$= 4330.13 \times 150 + 2500 \times 300$$
$$= 1399.52 \times 10^3 \text{ N-mm}$$

$$\sigma_b = \frac{M_b y}{I}$$

$$= \frac{(1399.52 \times 10^3)(t)}{\left[\frac{1}{12}(t)(2t)^3\right]} = \frac{2099.28 \times 10^3}{t^3} \text{ N/mm}^2$$

The direct tensile stress due to component P_h is given by,

$$\sigma_t = \frac{P_h}{A} = \frac{4330.13}{2t^2} = \frac{2165.07}{t^2} \text{ N/mm}^2$$

The vertical component P_v induces shear stress at the section XX. It is however small and neglected.