Example *A component machined from a plate made of steel 45C8 (S_{ut} = 630 N/mm^2) is shown in Fig. 5.29. It is subjected to a completely reversed axial force of 50 kN. The expected reliability is 90% and the factor of safety is 2. The size factor is 0.85. Determine the plate thickness t for infinite life, if the notch sensitivity factor is 0.8.*

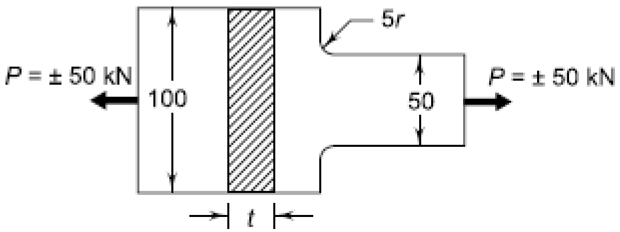


Fig. 5.29

Solution

Given $P = \pm 50$ kN $S_{ut} = 630$ N/mm^2 $(fs) = 2$
$R = 90\%$ $q = 0.8$ $K_b = 0.85$

Step I *Endurance limit stress for plate*

$S'_e = 0.5 S_{ut} = 0.5(630) = 315$ N/mm^2

From Fig. 5.24 (machined surface and S_{ut} = 630 N/mm^2),

$K_a = 0.76$

$K_b = 0.85$

For 90% reliability, $K_c = 0.897$

$$\left(\frac{D}{d}\right) = \frac{100}{50} = 2$$

and
$$\left(\frac{r}{d}\right) = \frac{5}{50} = 0.1$$

From Fig. 5.3, $K_t = 2.27$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.8\,(2.27 - 1) = 2.016$

$$K_d = \frac{1}{K_f} = \frac{1}{2.016} = 0.496$$

$$S_e = K_a K_b K_c K_d S'_e$$
$$= 0.76\,(0.85)(0.897)(0.496)(315)$$
$$= 90.54 \text{ N/mm}^2$$

Step II *Permissible stress amplitude*

From Eq. (5.30),

$$(S_e)_a = 0.8 S_e = 0.8(90.54) = 72.43 \text{ N/mm}^2$$

$$\sigma_a = \frac{(S_e)_a}{(fs)} = \frac{72.43}{2} = 36.22 \text{ N/mm}^2$$

Step III *Plate thickness*

Since $$\sigma_a = \frac{P}{(50t)}$$

$$\therefore \quad t = \frac{P}{50\sigma_a} = \frac{(50\times 10^3)}{50(36.22)} = 27.61 \text{ mm}$$

Example *A rotating bar made of steel 45C8 (S_{ut} = 630 N/mm²) is subjected to a completely reversed bending stress. The corrected endurance limit of the bar is 315 N/mm². Calculate the fatigue strength of the bar for a life of 90,000 cycles.*

Solution

Given $S_{ut} = 630\ \text{N/mm}^2$ $S_e = 315\ \text{N/mm}^2$ $N = 90000$ cycles

Step I *Construction of S–N diagram*

$0.9S_{ut} = 0.9\,(630) = 567\ \text{N/mm}^2$

$\log_{10}(0.9S_{ut}) = \log_{10}(567) = 2.7536$

$\log_{10}(S_e) = \log_{10}(315) = 2.4983$

$\log_{10}(90\,000) = 4.9542$

Also, $\log_{10}(10^3) = 3$ and $\log_{10}(10)^6 = 6$

Figure 5.30 shows the S–N curve for the bar.

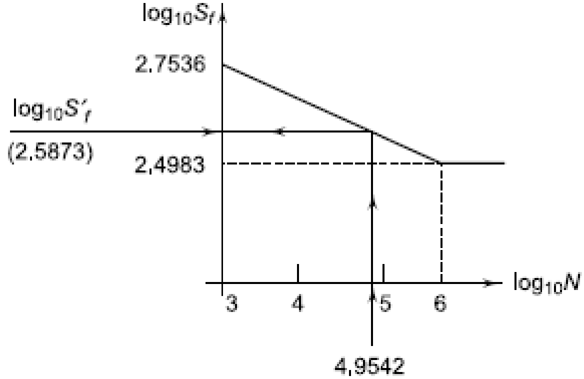


Fig. 5.30

Step II *Fatigue strength for 90000 cycles*

Referring to Fig. 5.30,

$$\log10\,(S_f') = 2.7536 - \frac{(2.7536 - 2.4983)}{(6-3)} \times (4.9542 - 3) = 2.5873$$

$$S_f' = 386.63\ \text{N/mm}^2$$

Example 5.7 *A forged steel bar, 50 mm in diameter, is subjected to a reversed bending stress of 250 N/mm². The bar is made of steel 40C8 (S_{ut} = 600 N/mm²). Calculate the life of the bar for a reliability of 90%.*

Solution

Given $S_f = \sigma_b = 250\ \text{N/mm}^2$

$S_{ut} = 600\ \text{N/mm}^2$ $R = 90\%$

Step I *Construction of S–N diagram*

$S_e' = 0.5S_{ut} = 0.5(600) = 300\ \text{N/mm}^2$

From Fig. 5.24, ($S_{ut} = 600\ \text{N/mm}^2$ and forged bar),

$K_a = 0.44$

For 50 mm diameter, $K_b = 0.85$

For 90% reliability, $K_c = 0.897$

$S_e = K_a K_b K_c S_e' = 0.44(0.85)\,(0.897)\,(300)$

$= 100.64\ \text{N/mm}^2$

$0.9S_{ut} = 0.9(600) = 540\ \text{N/mm}^2$

$\log_{10}(0.9S_{ut}) = \log_{10}(540) = 2.7324$

$\log_{10}(S_e) = \log_{10}(100.64) = 2.0028$

$\log 10(S_f) = \log_{10}(250) = 2.3979$

Also, $\log_{10}(10^3) = 3$ and $\log_{10}(10^6) = 6$

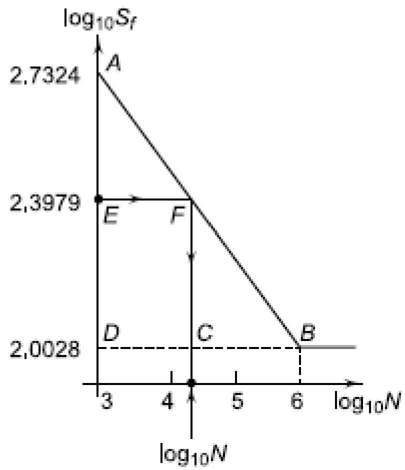
The S–N curve for the bar is shown in Fig. 5.31.

Step II *Fatigue life of bar*

From Fig. 5.31,

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.7324 - 2.3979)}{(2.7324 - 2.0028)}$$

$$= 1.3754$$



Therefore,

$\log_{10} N = 3 + \overline{EF} = 3 + 1.3754$

$\log_{10} N = 4.3754$

$N = 23\ 736.2$ cycles

Example *A rotating shaft, subjected to a non-rotating force of 5 kN and simply supported between two bearings A and E is shown in Fig. 5.32(a). The shaft is machined from plain carbon steel 30C8 (S_{ut} = 500 N/mm²) and the expected reliability is 90%. The equivalent notch radius at the fillet section can be taken as 3 mm. What is the life of the shaft?*

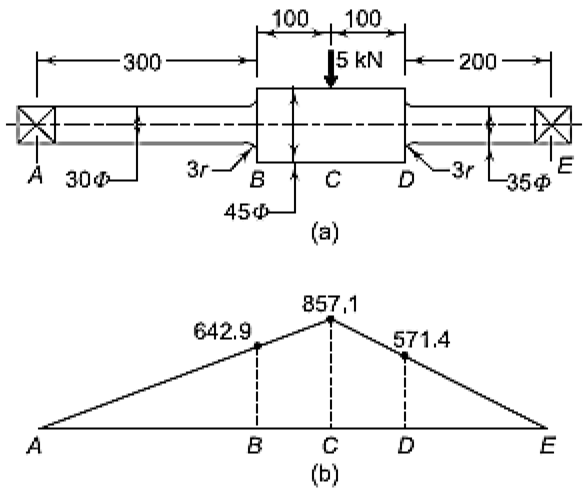


Fig. 5.32

Solution

Given $P = 5$ kN $\quad S_{ut} = 500$ N/mm²

$R = 90\%$ $\quad r = 3$ mm

Step I Selection of failure-section
Taking the moment of the forces about bearings A
and E, the reactions at A and E are 2143 and 2857 N
respectively. The bending moment diagram is shown
in Fig. 5.32(b). The values of the bending moment
shown in the fi gure are in N-m. The possibility of
a failure will be at the three sections B, C and D.
The failure will probably occur at the section B
rather than at C or D. At the section C, although the
bending moment is maximum, the diameter is more
and there is no stress concentration. At the section
D, the diameter is more and the bending moment is
less compared with that of section B. Therefore, it is
concluded that failure will occur at the section B.

***Step II** Construction of S–N diagram*

At the section B,

$$S_f = \sigma_b = \frac{32M_b}{\pi d^3} = \frac{32(642.9\times10^3)}{\pi(30)^3}$$

$$= 242.54 \text{ N/mm}^2$$

$$S_e' = 0.5S_{ut} = 0.5(500) = 250 \text{ N/mm}^2$$

From Fig. 5.24 (machined surface and S_{ut} = 500 N/mm²),

$$K_a = 0.79$$

For 30 mm diameter, $K_b = 0.85$

For 90% reliability, $K_c = 0.897$

Since $\dfrac{r}{d} = \dfrac{3}{30} = 0.1$ and $\dfrac{D}{d} = \dfrac{45}{30} = 1.5$

From Fig. 5.5, $K_t = 1.72$

From Fig. 5.22 (r = 3 mm and S_{ut} = 500 N/mm²),

$$q \simeq 0.78$$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.78\,(1.72 - 1)$
$= 1.5616$

$$K_d = \frac{1}{K_f} = \frac{1}{1.5616} = 0.64$$

$S_e = K_a\, K_b\, K_c\, K_d\, S_e'$

$= 0.79\,(0.85)(0.897)(0.64)(250) = 96.37$ N/mm²

$0.9S_{ut} = 0.9\,(500) = 450$ N/mm²

$\log_{10}(0.9S_{ut}) = \log_{10}(450) = 2.6532$

$\log_{10}(S_e) = \log_{10}(96.37) = 1.9839$

$\log_{10}(S_f) = \log_{10}(242.54) = 2.3848$

Also, $\log_{10}(10^3) = 3$ and $\log_{10}(10^6) = 6$

The S–N curve for the shaft is shown in Fig. 5.33.

Step III *Fatigue life of shaft*

From Fig. 5.33,

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.6532-2.3848)}{(2.6532-1.9839)}$$

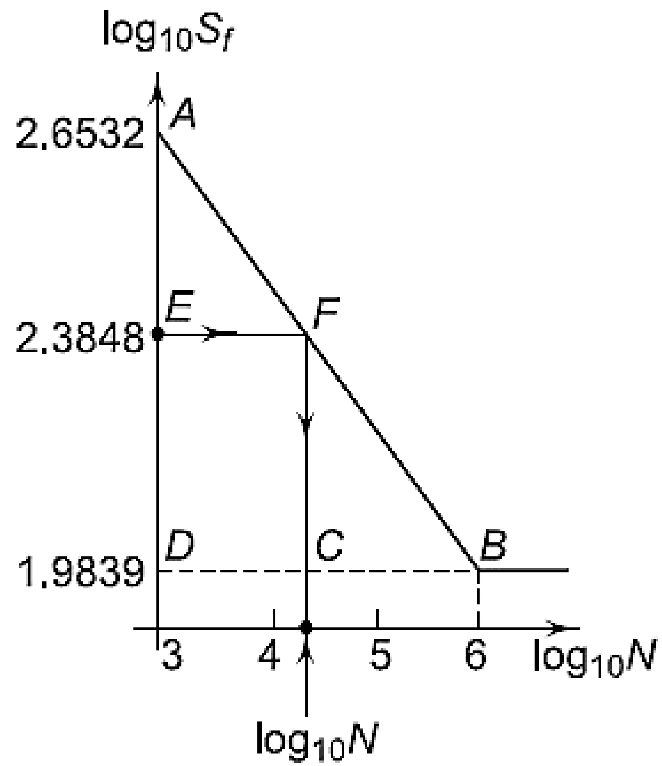
$$= 1.2030$$

Therefore,

$\log_{10} N = 3 + \overline{EF} = 3 + 1.2030$

$\log_{10} N = 4.2030$

$N = 15958.79$ cycles



- CUMULATIVE DAMAGE IN FATIGUE

In certain applications, the mechanical component is subjected to different stress levels for different parts of the work cycle. The life of such a component is determined by *Miner's equation*. Suppose that a component is subjected to completely reversed stresses (σ_1) for (n_1) cycles, (σ_2) for (n_2) cycles, and so on. Let N_1 be the number of stress cycles before fatigue failure, if only the alternating stress (σ_1) is acting. One stress cycle will consume ($1/N_1$) of the fatigue life and since there are n_1 such cycles at this stress level, the proportionate damage of fatigue life will be $[(1/N_1)n_1]$ or (n_1/N_1). Similarly, the proportionate damage at stress level (σ_2) will be (n_2/N_2). Adding these quantities, we get

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_x}{N_x} = 1 \qquad (5.33)$$

The above equation is known as Miner's equation. Sometimes, the number of cycles $n_1, n_2, \ldots$ at stress levels $\sigma_1, \sigma_2, \ldots$ are unknown. Suppose that $\alpha_1, \alpha_2, \ldots$ are proportions of the total life that will be consumed by the stress levels $\sigma_1, \sigma_2, \ldots$ etc. Let N be the total life of the component. Then,

$$n_1 = \alpha_1 N$$
$$n_2 = \alpha_2 N$$

Substituting these values in Miner's equation,

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \ldots + \frac{\alpha_x}{N_x} = \frac{1}{N} \qquad (5.34)$$

Also,

$$\alpha_1 + \alpha_2 + \alpha_3 + \ldots\ldots\ldots + \alpha_x = 1 \qquad (5.35)$$

With the help of the above equations, the life of the component subjected to different stress levels can be determined.