**Example** *A cantilever beam made of cold drawn steel 40C8 ($S_{ut}$ = 600 N/mm$^2$ and $S_{yt}$ = 380 N/mm$^2$) is shown in Fig. 5.42. The force P acting at the free end varies from –50 N to +150 N. The expected reliability is 90% and the factor of safety is 2. The notch sensitivity factor at the fillet is 0.9. Determine the diameter 'd' of the beam at the fillet cross-section.*

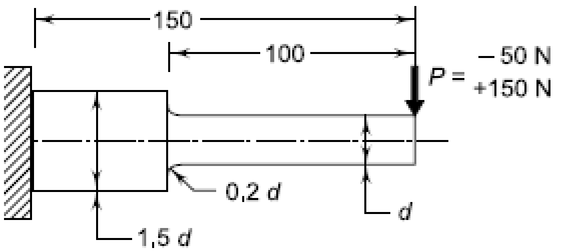


**Fig. 5.42**

**Solution**

**Given** $P = -50$ N to $+150$ N $\quad S_{ut} = 600$ N/mm$^2$ $\quad S_{yt} = 380$ N/mm$^2$ $\quad R = 90\%$ $\quad (fs) = 2$ $\quad q = 0.9$

***Step I*** *Endurance limit stress for cantilever beam*

$S'_e = 0.5 S_{ut} = 0.5\,(600) = 300$ N/mm$^2$

From Fig. 5.24 (cold drawn steel and $S_{ut}$ = 600 N/mm$^2$),

$$K_a = 0.77$$

Assuming $7.5 < d < 50$ mm,

$$K_b = 0.85$$

For 90% reliability, $K_c = 0.897$

Since, $\dfrac{r}{d} = 0.2$ and $\dfrac{D}{d} = 1.5$

From Fig. 5.5, $K_t = 1.44$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.9\,(1.44 - 1) = 1.396$

$$K_d = \frac{1}{K_f} = \frac{1}{1.396} = 0.716$$

$$S_e = K_a K_b K_c K_d S'_e = 0.77\,(0.85)\,(0.897)\,(0.716)\,(300) = 126.11 \text{ N/mm}^2$$

***Step II*** *Construction of modified Goodman diagram*

At the fillet cross-section,

$(M_b)_{max.} = 150 \times 100 = 15000$ N-mm

$(M_b)_{min.} = -50 \times 100 = -5000$ N-mm

$$(M_b)_m = \frac{1}{2}[(M_b)_{max.} + (M_b)_{min.}] = \frac{1}{2}[15000 - 5000] = 5000 \text{ N-mm}$$

$$(M_b)_a = \frac{1}{2}[(M_b)_{max} - (M_b)_{min.}] = \frac{1}{2}[15000 + 5000] = 10\,000 \text{ N-mm}$$

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m} = \frac{10000}{5000} = 2$$

$$\theta = 63.435°$$

The modified Goodman diagram for this example is shown in Fig. 5.43.

***Step III*** *Permissible stress amplitude*

Refer to Fig. 5.43. The coordinates of the point $X$ are determined by solving the following two equations simultaneously.

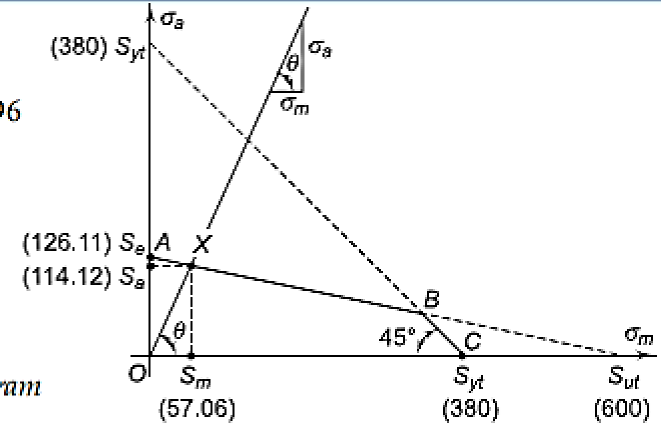


**Fig. 5.43**

(i) Equation of line $AB$

$$\frac{S_a}{126.11} + \frac{S_m}{600} = 1 \qquad \text{(a)}$$

(ii) Equation of line $OX$

$$\frac{S_a}{S_m} = \tan\theta = 2 \qquad \text{(b)}$$

Solving the two equations,

$S_a = 114.12$ N/mm$^2$ and $S_m = 57.06$ N/mm$^2$

***Step IV*** *Diameter of beam*

Since $\sigma_a = \dfrac{S_a}{(fs)}$ $\quad\therefore\quad \dfrac{32(M_b)_a}{\pi d^3} = \dfrac{S_a}{(fs)}$

$$\frac{32(10\,000)}{\pi d^3} = \frac{114.12}{2}$$

$$d = 12.13 \text{ mm}$$

- GERBER EQUATION
- The Soderberg line and Goodman line illustrated in
- Fig. 5.39 are straight lines. The theories using such
- straight lines for predicting fatigue failure are called
- ‘linear’ theories. There are some theories that use
- parabolic or elliptical curves instead of straight lines.
- These theories are called ‘non-linear’ theories. One
- of the most popular non-linear theories is the Gerber
- theory that is based on parabolic curve. The Gerber
- curve is shown in Fig. 5.52. The equation for the
- Gerber curve is as followsTheories based on the
- Soderberg line or the
- Goodman line, as failure criteria are conservative
- theories. This results in increased dimensions of the
- component. The Gerber curve takes the mean path
- through failure points. It is therefore more accurate
- in predicting fatigue failure.



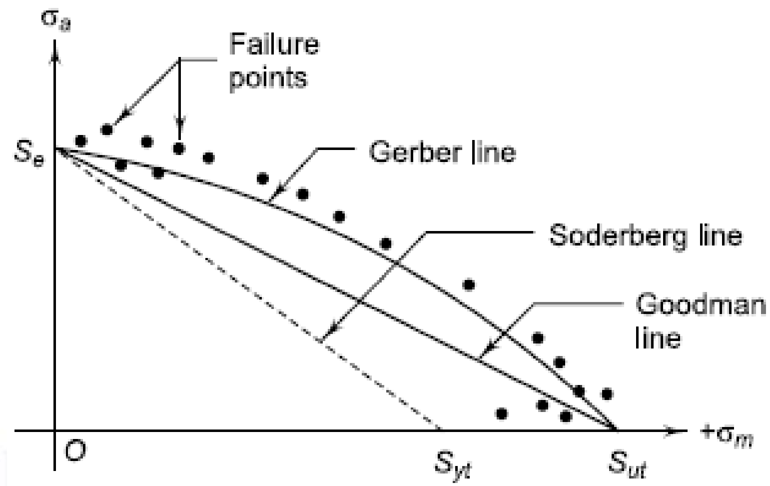


**Fig. 5.52** *Gerber line*

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1 \qquad (5.46)$$

The above equation is called the *Gerber equation*. It can be also written in the following form:

$$S_a = S_e\left[1 - \left(\frac{S_m}{S_{ut}}\right)^2\right] \qquad (5.47)$$

- Example 5 A machine component is subjected
- to fl uctuating stress that varies from 40 to 100
- N/mm2. The corrected endurance limit stress for
- the machine component is 270 N/mm2. The ultimate
- tensile strength and yield strength of the material
- are 600 and 450 N/mm2
- respectively. Find the factor
- of safety using(i) Gerber theory(ii) Soderberg line
- (iii) Goodman lineAlso, fi nd the factor of
- safety against staticfailure.

**Solution**

**Given** $S_{ut} = 600$ N/mm$^2$ $S_{yt} = 450$ N/mm$^2$
$S_e = 270$ N/mm$^2$ $\sigma_{max.} = 100$ N/mm$^2$
$\sigma_{min.} = 40$ N/mm$^2$

***Step I*** *Permissible mean and amplitude stresses*

$$\sigma_a = \frac{1}{2}(100-40) = 30 \text{ N/mm}^2$$

$$\sigma_m = \frac{1}{2}(100+40) = 70 \text{ N/mm}^2$$

$$S_a = n\sigma_a = 30\ n$$
$$S_m = n\sigma_m = 70\ n$$

where $n$ is the factor of safety.

***Step II*** *Factor of safety using Gerber theory*

From Eq. (5.46),

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

or

$$\left(\frac{30n}{270}\right) + \left(\frac{70n}{600}\right)^2 = 1$$

$$n^2 + 8.16\,n - 73.47 = 0$$

Solving the above quadratic equation,

$$n = 5.41 \qquad \text{(i)}$$

***Step III*** *Factor of safety using Soderberg line*

The equation of the Soderberg line is as follows,

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1$$

$$\left(\frac{30n}{270}\right) + \left(\frac{70n}{450}\right) = 1$$

$$n = 3.75 \qquad \text{(ii)}$$

***Step IV*** *Factor of safety using Goodman line*

The equation of the Goodman line is as follows:

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

$$\left(\frac{30n}{270}\right) + \left(\frac{70n}{600}\right) = 1$$

$$n = 4.39 \qquad \text{(iii)}$$

Step V Factor of safety against static failure

The factor of safety against static failure is given by,

$$n = \frac{S_{yt}}{\sigma_{max}} = \frac{450}{100} = 4.5$$

- **FATIGUE DESIGN UNDER COMBINED STRESSES**

- In practice, the problems are more complicated because the component may be subjected to two-dimensional stresses, or to combined bending and torsional moments. In case of two-dimensional stresses, each of the two stresses may have two components— mean and alternating. Similarly, the bending moment as well as torsional moment may have two components— mean and alternating. Such problems involving combination of stresses are solved by the distortion energy theory of failure.

$$\sigma^2 = \frac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)] \qquad \text{(a)}$$

where $\sigma_x$, $\sigma_y$, $\sigma_z$ are normal stresses in $X$, $Y$ and $Z$ directions and $\tau_{xy}$, $\tau_{yz}$, $\tau_{zx}$ are shear stresses in their respective planes. $\sigma$ is a stress which is equivalent to these three-dimensional stresses.

In case of two-dimensional stresses, the component is subjected to stresses $\sigma_x$ and $\sigma_y$ in $X$ and $Y$ directions.

Substituting $\sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$ in Eq. (a),

$$\sigma = \sqrt{(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2)} \qquad \text{(b)}$$

The mean and alternating components of $\sigma_x$ are $\sigma_{xm}$ and $\sigma_{xa}$ respectively. Similarly, the mean and alternating components of $\sigma_y$ are $\sigma_{ym}$ and $\sigma_{ya}$ respectively. In this analysis, the mean and alternating components are separately combined by Eq. (b), i.e.,

$$\sigma_m = \sqrt{(\sigma_{xm}^2 - \sigma_{xm}\sigma_{ym} + \sigma_{ym}^2)} \qquad (5.48)$$

Similarly,

$$\sigma_a = \sqrt{(\sigma_{xa}^2 - \sigma_{xa}\sigma_{ya} + \sigma_{ya}^2)} \qquad (5.49)$$

The two stresses $\sigma_m$ are $\sigma_a$ obtained by the above equations are used in the modified Goodman diagram to design the component.

In case of combined bending and torsional moments, there is a normal stress $\sigma_x$ accompanied by the torsional shear stress $\tau_{xy}$.

Substituting $\sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$ in Eq. (a),

$$\sigma = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \qquad \text{(c)}$$

The mean and alternating components of $\sigma_x$ are $\sigma_{xm}$ and $\sigma_{xa}$ respectively. Similarly, the mean and alternating components of $\tau_{xy}$ are $\tau_{xym}$ and $\tau_{xya}$ respectively. Combining these components separately by Eq. (c),

$$\sigma_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} \qquad (5.50)$$

$$\sigma_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} \qquad (5.51)$$

The two stresses $\sigma_m$ are $\sigma_a$ obtained by the above equations are used in the modified Goodman diagram to design the component.

- Example A machine component is subjected to two-dimensional stresses. The tensile stress in the X direction varies from 40 to 100 N/mm2 while the tensile stress in the Y direction varies from 10 to 80 N/mm2. The frequency of variation of these stresses is equal. The corrected endurance limit of the component is 270 N/mm2. The ultimate tensile strength of the material of the component is 660 N/mm2. Determine the factor of safety used by the designer.

**Solution**

**Given** $(\sigma_x)_{max.} = 100$ N/mm$^2$

$(\sigma_x)_{min.} = 40$ N/mm$^2$ $(\sigma_y)_{max.} = 80$ N/mm$^2$ $(\sigma_y)_{min.} = 10$ N/mm$^2$ $S_{ut} = 600$ N/mm$^2$ $S_e = 270$ N/mm$^2$

***Step I*** *Mean and amplitude stresses*

$$\sigma_{xm} = \frac{1}{2}(100 + 40) = 70 \text{ N/mm}^2$$

$$\sigma_{xa} = \frac{1}{2}(100 - 40) = 30 \text{ N/mm}^2$$

$$\sigma_{ym} = \frac{1}{2}(80 + 10) = 45 \text{ N/mm}^2$$

$$\sigma_{ya} = \frac{1}{2}(80 - 10) = 35 \text{ N/mm}^2$$

$$\sigma_m = \sqrt{(\sigma_{xm}^2 - \sigma_{xm}\sigma_{ym} + \sigma_{ym}^2)}$$

$$= \sqrt{[(70)^2 - (70)(45) + (45)^2]}$$

$$= 61.44 \text{ N/mm}^2$$

$$\sigma_a = \sqrt{(\sigma_{xa}^2 - \sigma_{xa}\sigma_{ya} + \sigma_{ya}^2)}$$

$$= \sqrt{[(30)^2 - (30)(35) + (35)^2]}$$

$$= 32.79 \text{ N/mm}^2$$

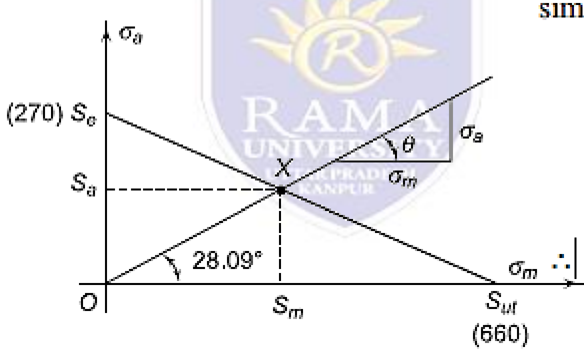


Fig. 5.55

***Step II*** *Construction of modified Goodman diagram*

$$\tan\theta = \frac{\sigma_a}{\sigma_m} = \frac{32.79}{61.44} = 0.534 \text{ or } \theta = 28.09°$$

The modified Goodman diagram for this example is shown in Fig. 5.55.

***Step III*** *Permissible stress amplitude*

Refer to Fig. 5.55. The co-ordinates of the point $X$ are obtained by solving the following two equations simultaneously.

$$\frac{S_a}{270} + \frac{S_m}{660} = 1 \qquad \text{(a)}$$

$$\frac{S_a}{S_m} = \tan\theta = 0.534 \qquad \text{(b)}$$

$$\therefore \quad S_a = 152.88 \text{ N/mm}^2$$

$$S_m = 286.29 \text{ N/mm}^2$$

***Step IV*** *Factor of safety*

$$(fs) = \frac{S_a}{\sigma_a} = \frac{152.88}{32.79} = 4.66$$