Maximum and Minimum Speeds of Driven Shaft

We have discussed in the previous article that velocity ratio,

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} \quad \text{or} \quad \omega_1 = \frac{\omega.\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} \qquad ...(i)$$

The value of ω_1 will be maximum for a given value of α, if the denominator of equation *(i)* is m. This will happen, when

$$\cos^2\theta = 1, \quad i.e. \text{ when } \theta = 0°, 180°, 360° \text{ etc.}$$

∴ Maximum speed of the driven shaft,

$$\omega_{1(max)} = \frac{\omega\cos\alpha}{1-\sin^2\alpha} = \frac{\omega\cos\alpha}{\cos^2\alpha} = \frac{\omega}{\cos\alpha} \qquad ...(ii)$$

$$N_{1(max)} = \frac{N}{\cos\alpha} \qquad ...(\text{where } N \text{ and } N_1 \text{ are in r.p.m.})$$

Similarly, the value of ω_1 is minimum, if the denominator of equation *(i)* is maximum. This pen, when ($\cos^2\theta$. $\sin^2\alpha$) is maximum, or

$$\cos^2\theta = 0, \ i.e. \text{ when } \theta = 90°, 270° \text{ etc.}$$

∴ Minimum speed of the driven shaft,

$$\omega_{1\,(min)} = \omega\cos\alpha$$

$$N_{1\,(min)} = N\cos\alpha \qquad ...(\text{where } N \text{ and } N_1 \text{ are in r.p.m.})$$

Condition for Equal Speeds of the Driving and Driven Shafts

We have already discussed that the ratio of the speeds of the driven and driving shafts is

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} \qquad \text{or} \qquad \omega = \frac{\omega_1(1-\cos^2\theta.\sin^2\alpha)}{\cos\alpha}$$

For equal speeds, $\omega = \omega_1$, therefore

$$\cos\alpha = 1-\cos^2\theta\;.\;\sin^2\alpha \qquad \text{or} \qquad \cos^2\theta\;.\;\sin^2\alpha = 1-\cos\alpha$$

$$\cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha} \qquad \ldots$$

We know that $\sin^2\theta = 1-\cos^2\theta = 1-\dfrac{1-\cos\alpha}{\sin^2\alpha} = 1-\dfrac{1-\cos\alpha}{1-\cos^2\alpha}$

$$= 1-\frac{1-\cos\alpha}{(1+\cos\alpha)(1-\cos\alpha)} = 1-\frac{1}{1+\cos\alpha} = \frac{\cos\alpha}{1+\cos\alpha} \qquad \ldots$$

Dividing equation (*ii*) by equation (*i*),

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos\alpha}{1+\cos\alpha} \times \frac{\sin^2\alpha}{1-\cos\alpha}$$

$$\tan^2\theta = \frac{\cos\alpha\,\sin^2\alpha}{1-\cos^2\alpha} = \frac{\cos\alpha.\sin^2\alpha}{\sin^2\alpha} = \cos\alpha$$

$$\therefore \qquad \tan\theta = \pm\sqrt{\cos\alpha}$$

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In the limit, when $\delta t \to 0$,

$$\alpha_t = \underset{\delta t \to 0}{\mathrm{Lt}} \left(\frac{\delta\omega}{\delta t} \right) = \frac{d\omega}{dt}$$

Component of angular acceleration in the direction perpendicular to ox,

$$\alpha_c = \frac{rx'}{\delta t} = \frac{ox' \sin \delta\theta}{\delta t} = \frac{(\omega + \delta\omega) \sin \delta\theta}{\delta t} = \frac{\omega \sin \delta\theta + \delta\omega . \sin \delta\theta}{\delta t}$$

Since $\delta\theta$ in very small, therefore substituting $\sin \delta\theta = \delta\theta$, we have

$$\alpha_c = \frac{\omega . \delta\theta + \delta\omega . \delta\theta}{\delta t} = \frac{\omega . \delta\theta}{\delta t}$$

...(Neglecting $\delta\omega . \delta\theta$, being very small)

In the limit when $\delta t \to 0$,

$$\alpha_c = \underset{\delta t \to 0}{\mathrm{Lt}} \frac{\omega . \delta\theta}{\delta t} = \omega \times \frac{d\theta}{dt} = \omega . \omega_P \qquad \ldots \left(\text{Substituting } \frac{d\theta}{dt} = \omega_P \right)$$

$\therefore$ Total angular acceleration of the disc

$= \text{vector } xx' = \text{vector sum of } \alpha_t \text{ and } \alpha_c$

$$= \frac{d\omega}{dt} + \omega \times \frac{d\theta}{dt} = \frac{d\omega}{dt} + \omega . \omega_P$$

where dθ/dtis the angular velocity of the axis of spin about a certain axis, which is perpendicular to the plane in which the axis of spin is going to rotate. This angular velocity of the axis of spin (i.e. dθ/dt) is known as angular velocity of precession and is denoted by ωP

. The axis, about which the axis of spin is to turn, is known as axis of precession. The angular motion of the axis of spin about the axis of precession is known as precessional angular motion

Gyroscopic Couple

Consider a disc spinning with an angular velocity ωrad/s about the axis of spin OX, in

anticlockwise direction when seen from the front, as shown in Fig. 14.2 (a). Since the plane in which

the disc is rotating is parallel to the plane YOZ,therefore it is called plane of spinning.