

The plane

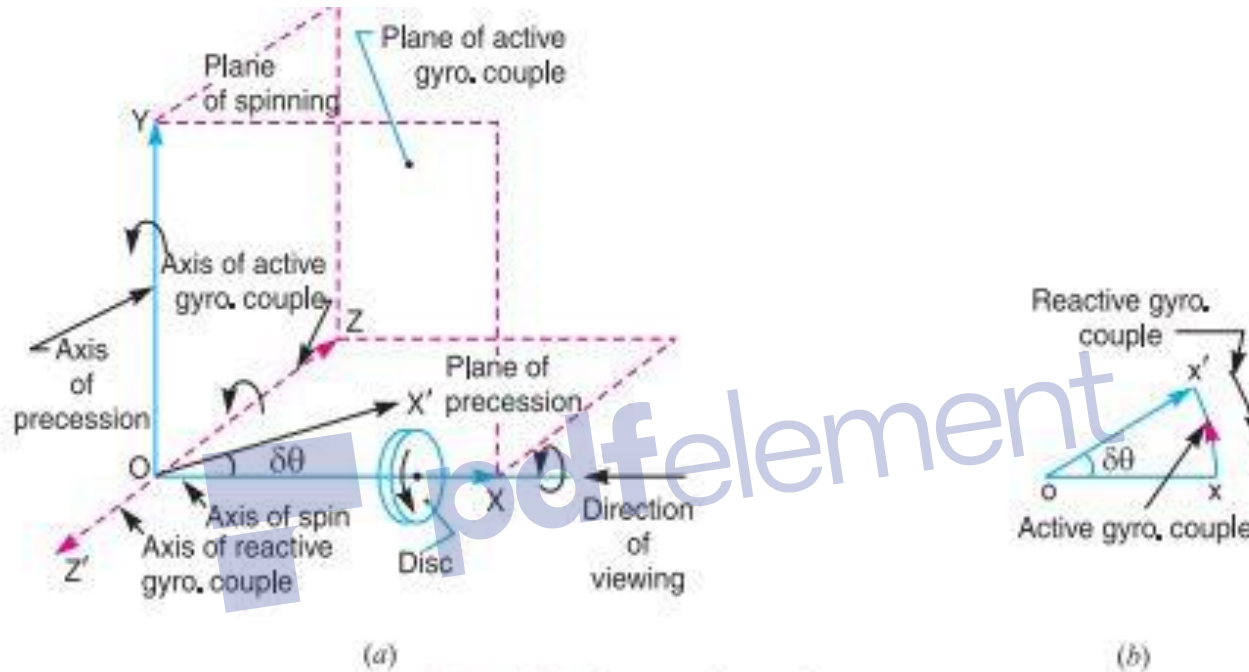
XOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY (which is perpendicular to both the axes OX and OZ) at an angular velocity  $\omega_P$  rad/s. This horizontal plane XOZ is called plane of precession and OY is the axis of precession.

Let  $I$  = Mass moment of inertia of the disc about OX, and  
 $\omega$  = Angular velocity of the disc.

$\therefore$  Angular momentum of the disc  $= I \cdot \omega$

Since the angular momentum is a vector quantity, therefore it may be represented by the vector  $ox$

as shown in Fig. (b). The axis of spin OX is also rotating anticlockwise when seen from the top about the axis OY. Let the axis OX is turned in the plane XOZ through a small angle  $\delta\theta$  radians to the position OX', in time  $\delta t$  seconds. Assuming the angular velocity  $\omega$  to be constant, the angular momentum will now be represented by vector  $ox$



where  $\omega_P$

= Angular velocity of precession of the axis of spin or the speed of rotation of the axis of spin about the axis of precession  $OY$ .

In S.I. units, the units of  $C$  is  $N\cdot m$  when  $I$  is in  $kg\cdot m^2$

It may be noted that

1. The couple  $I\omega\omega_p$ , in the direction of the vector  $xx'$  (representing the change in angular momentum) is the active gyroscopic couple, which has to be applied over the disc when the axis of spin is made to rotate with angular velocity  $\omega_p$  About the axis of precession. The vector  $xx'$  lies in the plane  $XOZ$  or the horizontal plane. In case of a very small displacement  $\delta\theta$ , the vector  $xx'$  will be perpendicular to the vertical plane  $XOY$ . Therefore the couple causing this change in the angular momentum will lie in the plane  $XOY$ . The vector  $xx'$ , as shown in Fig. (b), represents an anticlockwise couple in the plane  $XOY$ . Therefore, the plane  $XOY$  is called the plane of active gyroscopic couple and the axis  $OZ$  perpendicular to the plane  $XOY$ , about which the couple acts, is called the axis of active gyroscopic couple.
2. When the axis of spin itself moves with angular velocity  $\omega_p$ , the disc is subjected to reactive couple whose magnitude is same (i.e.  $I\omega\omega_p$ ) but opposite in direction to that of active couple.



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This reactive couple to which the disc is subjected when the axis of spin rotates about the axis of precession is known as reactive gyroscopic couple. The axis of the reactive gyroscopic couple is represented by  $OZ'$  in Fig.(a).

3. The gyroscopic couple is usually applied through the bearings which support the shaft. The bearings will resist equal and opposite couple.

4. The gyroscopic principle is used in an instrument or toy known as gyroscope. The gyroscopes are installed in ships in order to minimize the rolling and pitching effects of waves. They are also used in aeroplanes, monorail cars, gyrocompasses etc

**Question-**A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis?