12.11. Involute Teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig. 12.9. In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :

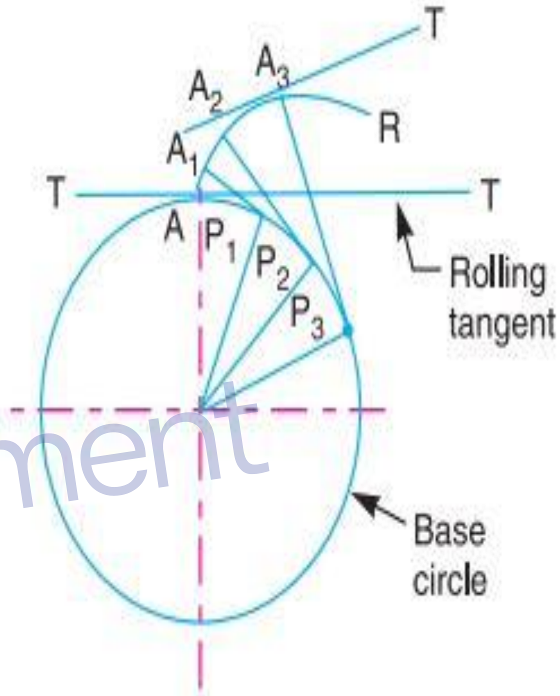

Fig. 12.9. Construction of involute.

Let A be the starting point of the involute. The base circle is divided into equal number of parts *e.g.* AP_1, P_1P_2, P_2P_3 etc. The tangents at P_1, P_2, P_3 etc. are drawn and the length P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc. we obtain the involute curve AR. A little consideration will show that at any instant A_3, the tangent A_3T to the involute is perpendicular to P_3A_3 and P_3A_3 is the normal to the involute. In other words, ***normal at any point of an involute is a tangent to the circle.***

Length of Arc of Contact

We have already defined that the arc of contact is the path traced by a point on the pitch circlefrom the beginning to the end of engagement of a given pair of teeth. In Fig, the arc of contactis EPFor GPH. Considering the arc of contact GPH, it is divided into two parts i.e. arc GPand arcPH. The arc GPis known as arc of approachand the arc PHis called arc of recess. The angles subtended by these arcs at O1 are called angle of approachand angle of recessrespectively.

We know that the length of the arc of approach (arc GP)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

and the length of the arc of recess (arc PH)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

Since the length of the arc of contact GPH is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact

$$= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch. Mathematically,

Contact ratio or number of pairs of teeth in contact

Length of the arc of contact c p = where Circular pitch , and c p m ==π Module. m=

Q-1 The number of teeth on each of the two equal spur gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular m pitch, find the addendum.

Solution. Given : $T = t = 40$; $\phi = 20°$; $m = 6$ mm

We know that the circular pitch,

$$p_c = \pi m = \pi \times 6 = 18.85 \text{ mm}$$

∴ Length of arc of contact

$$= 1.75\, p_c = 1.75 \times 18.85 = 33 \text{ mm}$$

d length of path of contact

$$= \text{Length of arc of contact} \times \cos\phi = 33 \cos 20° = 31 \text{ mm}$$

Let $R_A = r_A$ = Radius of the addendum circle of each wheel.

We know that pitch circle radii of each wheel,

$$R = r = m.T/2 = 6 \times 40/2 = 120 \text{ mm}$$

and length of path of contact

$$31 = \sqrt{(R_A)^2 - R^2 \cos^2\phi} + \sqrt{(r_A)^2 - r^2 \cos^2\phi} - (R + r)\sin\phi$$

$$= 2\left[\sqrt{(R_A)^2 - R^2 \cos^2\phi} - R\sin\phi\right] \quad ...(\because R = r, \text{ and } R_A = r_A)$$

$$\frac{31}{2} = \sqrt{(R_A)^2 - (120)^2 \cos^2 20^\circ} - 120 \sin 20^\circ$$

$$15.5 = \sqrt{(R_A)^2 - 12\,715} - 41$$

$$(15.5 + 41)^2 = (R_A)^2 - 12\,715$$

$$3192 + 12\,715 = (R_A)^2 \qquad \text{or} \qquad R_A = 126.12 \text{ mm}$$

We know that the addendum of the wheel,

$$= R_A - R = 126.12 - 120 = 6.12 \text{ mm}$$ **Ans.**

Q-2 A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

Length of path of contact

We know that pitch circle radius of pinion,

$$r = m.t/2 = 12 \times 30/2 = 180 \text{ mm}$$

and pitch circle radius of gear,

$$R = m.T/2 = 12 \times 80/2 = 480 \text{ mm}$$

$\therefore$ Radius of addendum circle of pinion,

$$r_A = r + \text{ Addendum} = 180 + 10 = 190 \text{ mm}$$

and radius of addendum circle of gear,

$$R_A = R + \text{Addendum} = 480 + 10 = 490 \text{ mm}$$

We know that length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2\phi} - R\sin\phi$$

$$= \sqrt{(490)^2 - (480)^2 \cos^2 20°} - 480 \sin 20°$$

and length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2\phi} - r\sin\phi$$

$$= \sqrt{(190)^2 - (180)^2 \cos^2 20°} - 180 \sin 20° =$$

We know that length of path of contact,

$$KL = KP + PL = 27.3 + 25 = 52.3 \text{ mm} \quad \textbf{Ans.}$$